Attenuation of a strong shock wave in a two-phase medium

V. M. Kudinov, B. I. Palamarchuk, and V. A. Vakhnenko

E. O. Paton Institute of Electric Welding, Academy of Sciences of the Ukrainian SSR, Kiev

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It is well known that the rate of attenuation of shock waves generated by pulsed energy sources in aerosols and foams is higher than in air,[1-3] and in bubbly media it is higher than in water.[4,5] Sedov et al.[1,2] have derived relations determining the parameters of a shock wave for thermodynamic equilibrium between the phases[1] and for a relaxing medium[2] from the solution of the problem of the strong stage of an explosion in a two-phase medium, assuming that the volume fraction ε of the condensed phase is small. A solution of the problem in the case of an arbitrary content ε is given in Refs. 6 and 7. However, in Ref. 6 an error in the equation for the internal energy of the medium limits the applicability of the results for large volume contents of the condensed phase. The results of Ref. 7, which were obtained by numerical methods, do not give the explicit dependence of the parameters of the shock wave on ε and cannot be used to investigate the influence of relaxation processes on its attenuation.

For the analysis of these problems we consider a strong shock wave in a homogeneous two-phase medium consisting of a condensed phase and a gaseous phase uniformly distributed in a volume with an arbitrary content of the incompressible condensed component. Within the framework of the customary assumptions[6-8] the equation of state of the two-phase medium is conveniently written in the form

$$E = \frac{p(1-\epsilon)}{\rho(\Gamma-1)}, \tag{1}$$

where Γ is a parameter that establishes a relation of the form (1) in a definite range of variation of the thermodynamic parameters of the mixture; the rest of the notation is conventional. In the general case Γ is a function of the relaxation of the parameters behind the wave front, and coincides with the adiabatic exponent γ of the gaseous phase for frozen processes and with the adiabatic exponent Γ_0 of the two-phase mixture for thermodynamic equilibrium.[9]

In the analysis, for definiteness we consider the process of attenuation of a strong shock wave for a constant Γ. Let an instantaneous release of energy E_0 take place in the two-phase medium at a plane ($\nu = 1$), a line ($\nu = 2$), or a point ($\nu = 3$), depending on the symmetry of the problem. Under these assumptions the solution of the problem is self-similar, i.e., the variables $R = \rho/\rho_0$, $V = u/D$, and $P = p/\rho_0 D^2$ do not depend on the time, in the region where the internal energy of the medium can be neglected in comparison with E_0. In this case they satisfy the system of equations[7]

$$(V-\eta)\frac{dR}{d\eta} + \frac{R}{\eta^{\nu-1}}\frac{d\eta^{\nu-1}V}{d\eta} = 0,$$

$$(V-\eta)\frac{dV}{d\eta} - \frac{\nu}{2}V + \frac{1}{R}\frac{dP}{d\eta} = 0,$$

$$(1-\epsilon_0 R)(V-\eta)\frac{dP}{d\eta} - \nu P(1-\epsilon_0 R) + \frac{\Gamma P}{\eta^{\nu-1}}\frac{d\eta^{\nu-1}V}{d\eta} = 0 \tag{2}$$

with the boundary conditions $V = 0$ for $\eta = 0$ and $\eta = 1$,

$$V = P = \frac{2(1-\epsilon_0)}{\Gamma-1}, \quad R = \frac{\Gamma+1}{\Gamma-1+2\epsilon_0} \tag{3}$$

(Rankine–Hugoniot conditions for a strong shock wave).

These equations are complemented with the integral relation

$$E_0 = \sigma(\nu)\rho_0 D^2 r_f^\nu \int_0^1 \left(\frac{P(1-\epsilon_0 R)}{\Gamma-1} + R\frac{V^2}{2}\right)\eta^{\nu-1}d\eta, \tag{4}$$

where $\eta = r/r_f$, and $\sigma(\nu) = 2(\nu-1)\pi + (\nu-2)(\nu-3)$; D and r_f are the velocity and spatial coordinate of the shock-wave front. The subscript 0 refers to the parameters ahead of the shock-wave front.

The volume of the incompressible condensed phase can be eliminated from the equations by means of a transformation, i.e., a coordinate system can be determined in which the motion of the two-phase medium is similar to the motion of an ideal gas. In this coordinate system the transformed variables R', V' and P' satisfy the well known system of equations describing a strong shock wave in a gas.[1,10] Here the single-valued relationship between the systems of equations is determined by the relations

$$\begin{aligned} &\eta\mu = \eta'(1 + \epsilon_0(\mu^2 R' - 1)) - \epsilon_0 \mu^2 V' R', \\ &V' = \mu\frac{V}{1-\epsilon_0}, \quad P' = \frac{P}{1-\epsilon_0}, \\ &R' = \mu^{-2} R\frac{1-\epsilon_0}{1-\epsilon_0 R}, \quad \mu = \left(\frac{\eta}{\eta'}\right)^{\nu-1}. \end{aligned} \tag{5}$$

The resulting transformation can be used to draw a complete analogy, in the primed coordinate system, between the motion of the two-phase medium and that of an ideal gas, not only for the case in which ε_0 is assumed to be small[11] but also in the general case with arbitrary ε_0. In particular, it is now possible to find R', V' and P' from the known results of the self-similar solution of the problem and tabulated data,[1,10,12] and also to use the algebraic relations (5) in order to find the true distributions of R, V, and P.

The influence of the volume fraction ε_0 of the condensed phase on the laws of motion of the shock wave and the values of the parameters at the shock front can be established analytically without having to find the distribu-

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tions of R, V, and P. For this purpose we transform Eq. (4) by means of (5):

$$E_0 = \sigma(\nu)\rho_0 D^2 r_f^\nu \frac{(1-\epsilon_0)^2}{\Gamma-1} \int_0^1 \left(P' + \frac{\Gamma-1}{2} R'(V')^2\right)(\eta')^{\nu-1} d\eta'. \quad (6)$$

Using dimensional methods,[1] we obtain equations for the trajectory of the shock wave:

$$r_f = \left(\frac{E_0}{\alpha\rho_0}\right)^{1/(\nu+2)} \left(\frac{t}{1-\epsilon_0}\right)^{2/(\nu+2)},$$
$$D = \frac{2}{(\nu+2)(1-\epsilon_0)} \left(\frac{E_0}{\alpha\rho_0}\right)^{0.5} r_f^{-\nu/2}, \quad (7)$$

where

$$\alpha = \frac{4\sigma\psi}{(\nu+2)^2(\Gamma-1)}, \quad \psi = \int_0^1 \left(P' + \frac{\Gamma-1}{2} R'(V')^2\right)(\eta')^{\nu-1} d\eta'. \quad (8)$$

We devote special attention to the integral ψ. It has been shown[2,6] that as $\Gamma \to 1$ the expression $[2/\Gamma+1)]\psi$ tends to a finite limit and for $\Gamma = 1$ its value is equal to $(2\nu)^{-1}$. If we calculate ψ from existing analytical and tabulated data (e.g., from Ref. 12), we find that in the range of variation of Γ from 1.1 to 1.4 the value of the integral differs from its limiting value ψ ($\Gamma = 1$) only by 1.4, 2.1, and 2.6% for planar, cylindrical, and spherical symmetry respectively. With this accuracy the equality (6) can be written in the form

$$E_0(\Gamma - 1) = \frac{\sigma(\nu)}{2\nu} \rho_0 r_f^\nu D^2 (1-\epsilon_0)^2. \quad (9)$$

Using the relations (3) and (7)-(9), we obtain the relationship of the pressure at the shock-wave front to the distance from the center of symmetry:

$$p = \frac{2(1-\epsilon_0)}{\Gamma+1} \rho_0 D^2 = \frac{4\nu}{\sigma} \frac{\Gamma-1}{\Gamma+1} \frac{E_0}{1-\epsilon_0} r_f^{-\nu}. \quad (10)$$

It follows from Eqs. (10) that at a fixed distance from the center of symmetry the minimum pressure will occur in a medium having the maximum shock compressibility of the gaseous phase [determined by the expression $(\Gamma + 1)/(\Gamma - 1)$] for the minmum volume content ε_0 of the condensed phase.

The pressure-attenuation coefficient at the shock-wave front, defined as the ratio of the pressure p_g in the pure gas to the pressure p_t in the two-phase medium, has the form

$$\frac{p_g}{p_t} = (1-\epsilon_0) \frac{\gamma-1}{\gamma+1} \frac{\Gamma+1}{\Gamma-1}. \quad (11)$$

An analysis of the attenuation coefficients in two limiting cases – for a frozen temperature of the condensed phase ($\Gamma = \gamma$) and for thermodynamic equilibrium between the phases ($\Gamma = \Gamma_0$; for two-phase media Γ_0 is close to unity) – indicate the possibility of either decreasing the parameters of shock waves or increasing them, depending on the volume fraction of the condensed phase, the thermophysical properties of the phases, and the completeness of the relaxation process.

As is known, in the general case the parameters of shock waves from real energy sources for nonrelaxing media such as a gas are close to the parameters of a shock wave from a point source in the narrow region bounded by the distances at which the waves were generated, but which is still strong. A comparison of the experimental pressure-attenuation coefficients in the indicated region for foams[8] and bubbly media[5] and also of the normal stress for soils[13] and porous materials[14] with the calculated limiting values leads to the following conclusions:

1) All the experimental attenuation coefficients lie in the domain bounded by the parameters $\Gamma = \gamma$ and $\Gamma = \Gamma_0$, indicating appreciable nonequilibrium behind the shock-wave front in the indicated two-phase media.

2) For $\varepsilon_0 < 0.7$-0.8 (in this case the compressibility of the condensed phase does not introduce a perceptible contribution to the compressibility of the medium) the exponent s in the expression $p \sim r_f^{-s}$ for a spherical wave is close to $s \simeq \nu = 3$, and increases with the volume content of the gas.

Thus, for a correct description of the attenuation of shock waves in two-phase media (in the case when the compressibility of the medium is determined by the compressibility of the gaseous component) it is necessary to allow for the relaxational nature of the energy dissipation behind the shock-wave front. Allowance for this fact has made it possible, in particular, to obtain the true pattern of the attenuation of shock waves for gas–liquid foams[8] and to show that the observed increase of s in comparison with the self-similar solution is associated with relaxational heating of the condensed phase.

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