Tenth International Congress
on Sound and Vibration
7-10 July 2003 • Stockholm, Sweden

THE WAVE SPECTRAL EVOLUTION IN A DISCRETE MEDIUM WITH NONLINEARITY

Sergiy Mykulyak, Vyacheslav Vakhnenko, Vyacheslav Danylenko

Institute of Geophysics, Department of Dynamics of Nonhomogeneous Media
32 Palladin Avenue,
03680 Kyiv, Ukraine
e-mail: mikulyak@ukr.net

Abstract
The computer modelling of the propagation of weak nonlinear waves in a discrete medium is carried out. The structured medium is considered to be a massif of three sizes spheres. The separated spheres interact each other according to the Hertz law. The spectral analysis of the wave perturbation has shown that this model medium possesses the nonlinear properties. The nonlinearity is caused by both the nonlinear contacts of the discrete elements and the redistribution of the energy between different scale elements. Under the wave propagation, the relative part of the high frequencies in wave increases.

INTRODUCTION

Nonlinear elastic response appears to place it in a broader class of geological materials. The extensive literature on experimental studies of the nonlinear behaviour of different rocks is reviewed in [1]. Nonlinear response may manifest itself in a variety of manners, such as a nonlinear stress strain relations, nonlinear attenuation, harmonic generation and resonant peak shift [2-5]. Furthermore, rocks exhibit another unusual elastic properties that are beyond classical elastic behaviour: hysteresis and discrete memory in quasi-static deformation [6], slow dynamics [7-8]. All these non-classic properties result from the availability the mesoscopic structure. For example, the sandstone is the complicated hierarchic aggregate of constrained grains [9]. At the same time, the developed theoretical models of rocks [1] describe only some feature of dynamic and quasi-static behaviour of ones but not all. The classical model, created using the continual approach, is successful only for studying nonlinear wave propagations [10]. The discrete model called the Preisach-Mayergosz Space Model [11] quite good describes the static and dynamic nonlinearity, hysteresis and discrete memory. Nevertheless this model remains a phenomenological description, which does not contain the physical mechanisms of nonlinear response. More advanced model is one proposed by Guier R.A. and Johnson P.A. in [12], which considers the rock as a system of rigid particles bonded together by elastically soft amorphous material called “bond system”. In this report we suggest to consider the bond system as a lower level of hierarchy in comparison with the first level of rigid particles, therefore it consists of rigid particles with smaller sizes. Within this model we have carried out the computer simulation of the nonlinear wave propagation and the evolution of wave spectrum.

MATHEMATICAL MODEL

We consider a structured geophysical material to be a discrete hierarchic system. The rigid particles (blocks) at all levels have the spherical forms that enable us to save a great computer resource. This idealization is not principal because we will restrict our consideration by studying only the nonlinear wave propagation. The particles interact each other by the Hertzian contact law, while attraction between the blocks is neglected. We restrict our modelling to three levels of the hierarchy, i.e. we consider the massif of blocks, which is formed by three ensembles of particles with the identical sizes inside each ensemble. The interaction force between blocks depends on the nature of surfaces of these blocks as well as on the closing of the block centres. For i-th and j-th blocks the value of closing of blocks δ_{ij} is calculated as

$$\delta_{ij} = 2r - \sqrt{\sum_{k=1,2} (x_i^k - x_j^k)^2} \, ,$$

where x_i^k, x_j^k are coordinates of the centres of i-th and j-th blocks. The force $\mathbf{F}_{ij}$ can be expanded in the force $\mathbf{F}_{ij}^n$ acting along the line connecting two block centres and the force $\mathbf{F}_{ij}^{s}$, which is directed perpendicular to this line. The force $\mathbf{F}_{ij}^{n}$ depends on the value δ_{ij} nonlinearly

$$\mathbf{F}_{ij}^{n} = C_n \delta_{ij}^{\alpha} \mathbf{n}_{ij},$$

where C_n is a constant, which in accordance with Hertz law is defined by relation

$$C_n = \frac{\sqrt{2}\,E}{3(1-\nu^2)} \left(\frac{1}{r_i} + \frac{1}{r_j} \right)^{-1/2},$$

here E is Young's module, ν is Poisson's ratio, α=3/2, $\mathbf{n}_{ij}$ is the unit vector directed along the line between block centres. The tangent force $\mathbf{F}_{ij}^{s}$ depends on a relative shift in the direction perpendicular to the vector $\mathbf{n}_{ij}$ and is calculated as

$$\frac{d\mathbf{F}_{ij}^{s}}{dt} = -C_s \mathbf{w}_{ij}, \quad (1)$$

if $F_{ij}^{s} < C_k F_{ij}^{n}$. Otherwise, $\mathbf{F}_{ij}^{s}$ is calculated as

$$\mathbf{F}_{ij}^{s} = C_k \frac{\mathbf{w}_{ij}}{w_{ij}} F_{ij}^{n}. \quad (2)$$

In Eqs. (1) and (2) the value $\mathbf{w}_{ij}$ is the relative velocity of blocks i and j slippage:

$$\mathbf{w}_{ij} = \mathbf{v}_i - \mathbf{v}_j - \mathbf{n}_{ij}((\mathrm{v}_i - \mathrm{v}_j)\mathbf{n}_{ij}) + (2r - \delta_{ij})[\mathbf{n}_{ij} \times (\boldsymbol{\omega}_i \times \boldsymbol{\omega}_j)],$$

where $\mathbf{v}_i$ and $\boldsymbol{\omega}_i$ are the linear and the angle velocity of block i, respectively; constant C_s is the friction coefficient.

A motion of i-th block is given by the system of differential equations

$$m_i \frac{d^2\mathbf{x}_i}{dt^2} = \sum_j \mathbf{F}_{ij}, \quad (3)$$

$$I_i \frac{d^2\mathbf{\Phi}_i}{dt^2} = \sum_j \mathbf{M}_{ij}, \quad (4)$$

here $\mathbf{x}_i, \mathbf{\Phi}_\mathrm{i}, m_i, I_i$ are the radius-vector, the angle coordinate, the mass and the inertia moment of the block *I*; $\mathbf{M}_{ij}$ is the force moment acting on the block i from block j. The summarizing is carried out over all blocks j, which contact on block i. We consider 2D problem, since there is the limited computer resource. System of equations (1) - (4) is solved numerically using the Verlet algorithm [13, 14].

At the initial time, blocks are randomly located in the rectangular aria $0 \le x_1 \le L_1$, $0 \le x_2 \le L_2$. Filled in such manner the block massif is not compact packed, therefore it was initially consolidated. The packed massif contained 6720 particles is the subject for investigation of the nonlinear wave propagation. The system is set in motion by the piston driving at the initial time with the velocity V_0 and coordinate $x_1 = 0$ in x_1 direction. Defining the periodic conditions on boundaries $x_2 = 0$ and $x_2 = L_2$, the massif is not subjected to sided walls, therefore nonlinear wave can be considered to be plane. Taking into account that the motion of particles has a plain symmetry, we can calculate the averaged velocity as the velocity of a speculative surface, which moves in such a way that impulse fluxes from left side and right side on this surface are equal each other. This is the same as an infinitesimal flat plane would move together with the discrete medium. The calculation was stopped before the rarefaction wave formed in the right end of the massif reaches the last testing surface. In this way, we exclude any affect of the free boundary. The distribution of mass velocities over the particles as well as the location of tested surfaces is presented in Fig.1.

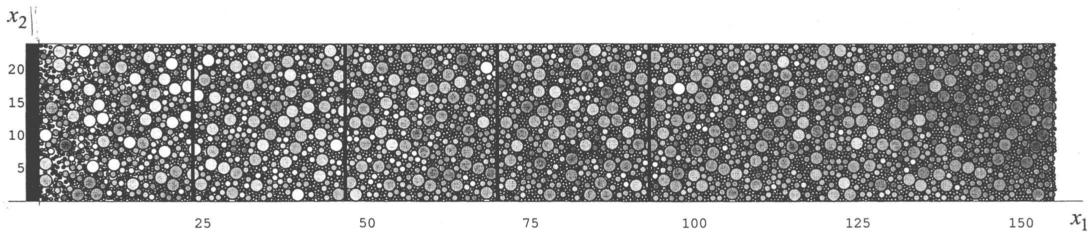

Fig.1. Velocity field $V(x_1, x_2)$ at the moment $t = 0.03$ sec. Dark colour presents larger velocity.

Figure 2 shows the time dependences of the averaged velocity at four different distances on coordinate origin. Spectra of these signals are presented in Fig.3. It is seen that spectra at various distances x_1 differ from each other: a part of high frequencies in spectrum rises along the distance x_1. It is necessary to note that the same feature has the spectrum of the wave under propagation in Borea

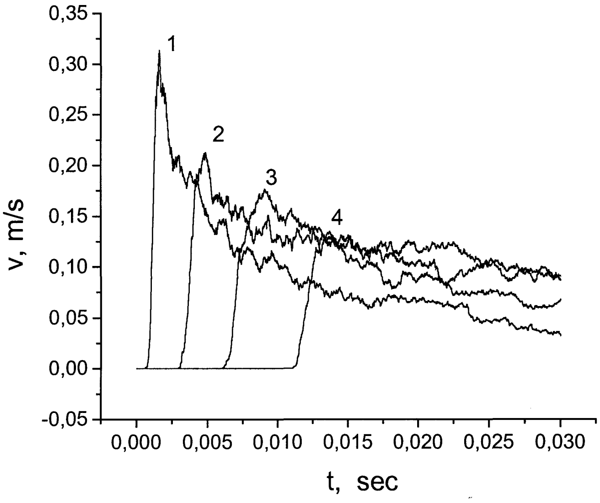

Fig.2. Averaged velocity dependences of time: (1) $x_1 = 23.2\,cm$*; (2)* $x_1 = 46.4\,cm$*; (3)* $x_1 = 69.7\,cm$*; (4)* $x_1 = 92.8\,cm$*.*

sandstone measured in experiments [10]. Thus, this model medium possesses

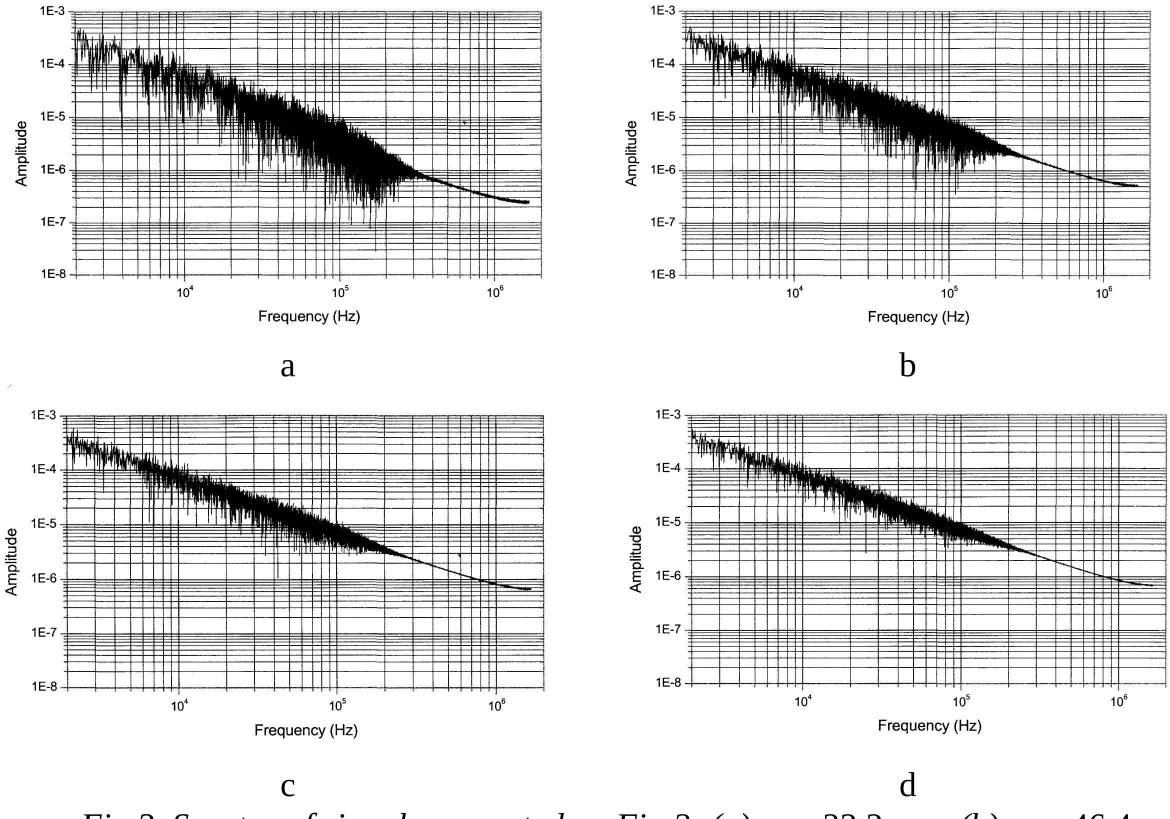

Fig.3. Spectra of signals presented on Fig.2: (a) $x_1 = 23.2\,cm$*; (b)* $x_1 = 46.4\,cm$*; (c)* $x_1 = 69.7\,cm$*; (d)* $x_1 = 92.8\,cm$*.*

a nonlinearity; and when appropriate parameters of the discrete medium are chosen, this model can be used to describe nonlinear wave propagation in rock.

CONCLUSION

We propose to consider rocks as discrete hierarchic organized system with the nonlinear interactions between elements of structure. The bond system is treated as the lower level of the hierarchic system. We have made the first step in modelling nonlinear response of rocks, being in the frames of this approach: we have studied the evolution the spectrum of wave, which propagates in model medium formed by the massif of spheres with different sizes. We have found that the spectrum of propagating wave shifts in the region of high frequencies like in laboratory experiments with sandstone. Thus, this model medium possesses nonlinearity and can be used to describe nonlinear wave propagation in rock. The study of dynamics of a discrete system for describing of hysteresis and discrete memory as well as slow dynamics, taking into account both repulsion and attraction as well as the nonspherical forms of the particles is in progress.

ACKNOWLEDGEMENT

The research was supported in part by STCU, project No 1747.

REFERENCES

1. L.A. Ostrovsky and P.A. Johnson, "Dynamic nonlinear elasticity in geomaterials", Rivista del nuovo cimento, **24**, No.7, 1-46 (2001)
2. K. Winkler, A. Nur and M. Gladwin, "Friction and seismic attenuation in rocks", Nature, **277**, 528-531 (1979)
3. V.N. Bakulin and A.G. Protosenya, "Nonlinear effects in travel of elastic waves through rocks", Transactions (Doklady) of the USSR Academy of Science. Earth Sciences Section, **263**, 314-316 (1981)
4. P.A. Johnson, T.J. Shankland, R.J. O'Connel and J.N. Albright "Nonlinear generation of elastic waves in crystalline rock", J. Geophys. Res., **92**, 3597-3602 (1987)
5. J.A. TenCate, K.A. Van Den Abeele, T.J. Shankland "Laboratory study of linear and nonlinear elastic pulse propagation in sandstone", J. Acoust. Soc. Am., **100**, No.3, 1383-1391 (1996)
6. R.A. Gueyer, K.R. McCall, G.N. Boitnott, L.B. Hilbert and T.J. Plona, "Quantitative use of Preisach-Maergoyz space to find static and dynamic elastic moduli in rock", J. Geophys. Res., **102**, 5281-5293 (1997)
7. J.A. TenCate, T.J Shankland, "Slow dynamics in the nonlinear elastic response of Berea sandstone", Geoph. Res. Lett., **23**, No. 21, 3019-3022 (1996)
8. R.A. Gueyer, P.A. Johnson, "Hysteresis, energy landscape and slow dynamics: a survey of the elastic properties of rocks", Journal of Materials Processing and Manufacturing Science, **9**, July, 14-26. (2000)
9. B. Zinszner, P.A. Johnson, and P.N.J. Rasolofosaon, "Influence of change in physical state on elastic nonlinear response in rock: effects of confining pressure and saturation" J. Geoph. Res., **102**, 8105-8120 (1997)

10. P.A. Johnson, K.R. McCall, “Observation and implications of nonlinear wave response in rock”, Geoph. Res. Let., **21**, No.3, 165-168 (1994)
11. K.R. McCall and P.A. Johnson, “Hysteresis, energy landscape and slow dynamics: a survey of the elastic properties of rocks”, Journal of Materials Processing and Manufacturing Science, **9**, July, 14-26 (2000)
12. P.A. Johnson, K.R. McCall, “Observation and implications of nonlinear wave response in rock”, Geoph. Res. Let., **21**, No.3, 165-168 (1994)
13. H.C. Andersen, “Molecular dynamics simulation at constant pressure and/or temperature”, J. Chem. Res., **72**, No.4, 2384-2393 (1980)
14. M.P. Allen and D.J. Tildesley, *Computer simulation of liquids*. (Claredon Press, Oxford, 1987)