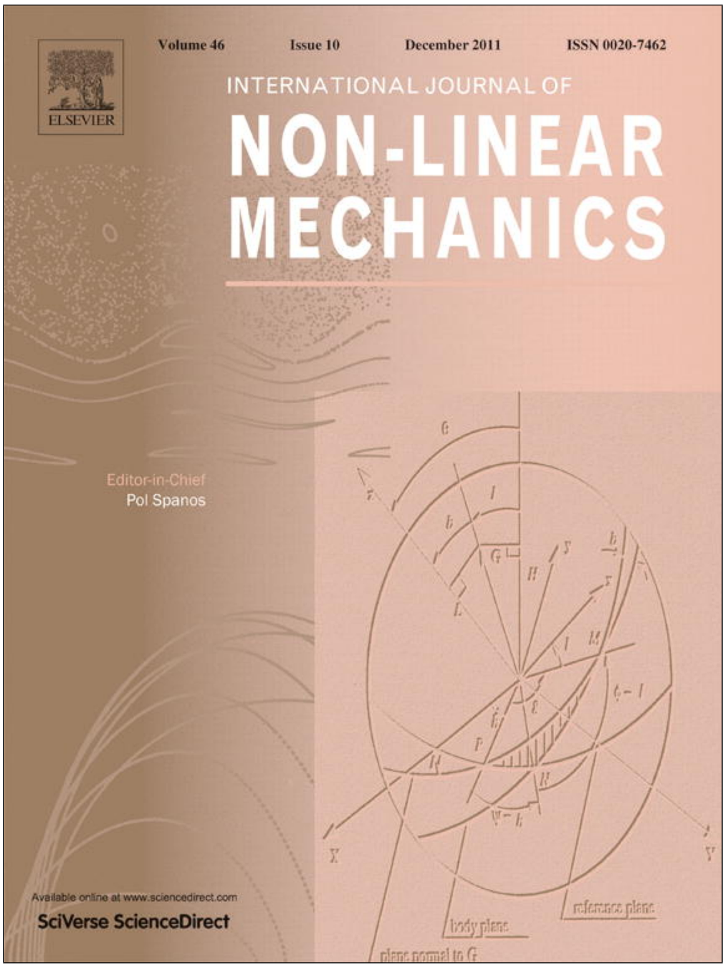
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Similarity in stationary motions of gas and two-phase medium with incompressible component

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ABSTRACT

In this paper we suggest the transformation between the equations for a perfect gas and the equations describing in one-velocity approach the two-phase medium with any volume occupied by the incompressible phase. It is proved that the motion of a two-phase medium in the transformed coordinate system is similar with certain accuracy to that of a perfect gas. It means that the solutions obtained for perfect gas can be used to solve wave problems for media with incompressible component. There is no necessity directly to solve the problem for medium with incompressible component, and it is only sufficient to transform the known solution of the similar problem for a homogeneous medium. Thus, the solutions of many hydrodynamic problems for multi-component media with incompressible phase can be obtained without solving the original set of equations. The scope for the suggested transformation is demonstrated by reference to the strong explosion in a two-phase medium.

1. Introduction

The paper deals with a comparison between the motion of a perfect gas and that of a two-phase medium with any volume occupied by the incompressible condensed component. Traditionally, it is regarded that in heterogeneous media with wavelength appreciably exceeding the size of the structural heterogeneities, the perturbations propagate in the same way as in homogeneous media [1,2]. In the framework of continuum mechanics [3] the idealization of a real medium as homogeneous has enabled considerable success for describing the wave processes (see Ref. [2] and references therein). It is known [1,2,4,5] that in one-velocity approach at low volume portion of the condensed phase ε, the motion of a two-phase medium is similar to the motion of a gas. For describing the motion of two-phase medium without restriction on a value of the volume portion ε, it is necessary to introduce this value ε as additional variable in the system of the hydrodynamic equations in contrast to the usual gas-dynamic equations. In approaches of other authors [6,7] such an extended system of equations have been treated by solving it separately for each particular ε.

We focus our attention on transformation between the system of three equations for a perfect gas and that for a two-phase medium with any volume occupied by the condensed phase. It shall allow one to apply the known solutions for perfect gas in order to solve the wave problems in two-component medium without solving the original system of equations.

Recently for planar motions only, we obtained the transformation between the systems of equations describing both these media in Eulerian coordinates [8,9]. In these papers it was shown that the motion of a two-component medium in the transformed coordinates is identical to the motion of a perfect gas. However, the transformation obtained in [8,9] reveals significant restriction, namely, for cylindrical and spherical symmetries the time varies not identically at all points in space.

2. System of equations in Lagrangian coordinates

We make the efforts to overcome the above restriction. In some sense the progress was achieved owing to the stimulating support of colleagues. One of a concept consists in analyzing the considered problem in the Lagrangian coordinates (ξ,τ). Let us consider a two-phase medium consisting of a condensed phase and a gaseous phase uniformly distributed in a volume. The incompressible condensed component can occupy an arbitrary partial-specific volume ε. We assume the following: (a) the condensed phase is incompressible; (b) the gas obeys the state equation for a perfect gas; (c) the partial pressure of the condensed phase is negligibly small; (d) the velocities of the condensed phase and gaseous phase equal each other. The conservation laws for mass, momentum, and energy give us the following system of the equations for the one-dimensional

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motions in the Lagrangian coordinates [3,10]:

$$\frac{r^{\nu-1}}{\xi^{\nu-1}}\left(\frac{\partial r}{\partial \xi}\right)_\tau=\frac{\upsilon}{\upsilon_0},\qquad u=\left(\frac{\partial r}{\partial \tau}\right)_\xi,$$

$$\left(\frac{\partial u}{\partial \tau}\right)_\xi+\upsilon_0\left(\frac{r}{\xi}\right)^{\nu-1}\left(\frac{\partial p}{\partial \xi}\right)_\tau=0,\qquad \frac{\partial E}{\partial \tau}+p\upsilon_0\xi^{1-\nu}\left(\frac{\partial r^{\nu-1}u}{\partial \xi}\right)_\tau=0. \quad (1)$$

Here υ is the specific volume. The parameter ν that determines the symmetry of the two-phase flow is equal to 1, 2 and 3 correspondingly for planar, cylindrical and spherical symmetries. The rest of the notations are conventional ones. Index 0 relates the variables to the unperturbed medium state. Note that the Eulerian space coordinate $r=r(\xi,\tau)$ is a dependent variable. Within the accepted assumptions the state equation for the two-phase medium is conveniently written in the form [4,7,11]

$$E=\frac{p\upsilon(1-\varepsilon)}{\gamma-1}. \quad (2)$$

Since the state equation (2) contains value of the volume portion as additional value

$$\varepsilon=\varepsilon_0\frac{\upsilon_0}{\upsilon}. \quad (3)$$

Eq. (2) does not coincides with state equation for a perfect gas with a certain effective adiabatic parameter γ. Considering the adiabatic flow γ to be constant, the equation for energy can be reduced to the form [10]:

$$\left(\frac{\partial p(\upsilon-\varepsilon_0\upsilon_0)^\gamma}{\partial \tau}\right)_\xi=0. \quad (4)$$

Thus, the closed system of the equations consists of first three equations of (1) and (3), (4).

Since the Eulerian space coordinate $r=r(\xi,\tau)$ is a dependent variable, we write this dependence through the Lagrangian independent coordinates (ξ,τ)

$$dr=\frac{\xi^{\nu-1}}{r^{\nu-1}}\frac{\upsilon}{\upsilon_0}d\xi+u\,d\tau. \quad (5)$$

We now show that for stationary motions as well as, with certain accuracy, for self-similar flows, one can find new variables in which all Eqs. (1), (3)–(4) coincide with equations for a perfect gas and are explicitly independent on ε.

The following physical backgrounds provide a basis for eliminating the volume portion ε from (1)–(4). Indeed, if the condensed phase does not vary its volume (condition (a)) and does not contribute into partial pressure (condition (c)) and moves along the paths of the compressible gaseous phase (condition (d)), then we can assume that eliminating of the volume occupied by the condensed phase ε should substantially simplify the mathematical description of motion.

3. Similarity of stationary flows

We need to reduce the system of Eqs. (1)–(4) to the system of equations describing the motion of a perfect gas (hereafter the notations for gas have primes)

$$\left(\frac{r'}{\xi'}\right)^{\nu-1}\left(\frac{\partial r'}{\partial \xi'}\right)_{\tau'}=\frac{\upsilon'}{\upsilon'_0},\qquad u'=\left(\frac{\partial r'}{\partial \tau'}\right)_{\xi'},$$

$$\left(\frac{\partial u'}{\partial \tau'}\right)_{\xi'}+\upsilon'_0\left(\frac{r'}{\xi'}\right)^{\nu-1}\left(\frac{\partial p'}{\partial \xi'}\right)_{\tau'}=0,\qquad \left(\frac{\partial p'(\upsilon')^\gamma}{\partial \tau'}\right)_{\xi'}=0. \quad (6)$$

For the latter system (6) the relation between the Eulerian space coordinate and the Lagrangian coordinates is as follows:

$$dr'=\left(\frac{\xi'}{r'}\right)^{\nu-1}\frac{\upsilon'}{\upsilon'_0}d\xi'+u'\,d\tau'. \quad (7)$$

One of the key requirement: the time should be equivalently running in all systems of coordinates $t=\tau=\tau'$.

The perturbations in incompressible component propagate with infinite velocity. Hence, the volume occupied by incompressible phase can be eliminated, then the connection between the equation (4) and the last equation (6) has the form

$$\upsilon'=\upsilon-\varepsilon_0\upsilon_0, \quad (8)$$

$$p'=p. \quad (9)$$

The relationship (8) indicates that the volume occupied by incompressible component is eliminated, and all masses of the medium are distributed over the residual volume of the compressible component.

Comparing the mass equations with each other, i.e. first equations from system (1) and system (6), the condition

$$\varepsilon_0+(1-\varepsilon_0)\left(\frac{r'}{\xi'}\right)^{\nu-1}\left(\frac{\partial r'}{\partial \xi'}\right)_{\tau'}=\left(\frac{r}{\xi}\right)^{\nu-1}\left(\frac{\partial r}{\partial \xi}\right)_\tau \quad (10)$$

should be satisfied.

We need also to make consistent the momentum equations (i.e. third equation in (1) and third equation in (6)), which after several reductions can obtain the form (see Appendix A):

$$\left(\frac{\partial u}{\partial \tau}\right)_\xi-\gamma p_0\left(\frac{r}{\xi}\right)^{\nu-1}\left(\frac{\upsilon'_0}{\upsilon'}\right)^{\gamma+1}\frac{1}{1-\varepsilon_0}\left(\frac{\partial \upsilon'}{\partial \xi}\right)_\tau=0, \quad (11)$$

$$\left(\frac{\partial u'}{\partial \tau'}\right)_{\xi'}-\gamma p'_0\left(\frac{r'}{\xi'}\right)^{\nu-1}\left(\frac{\upsilon'_0}{\upsilon'}\right)^{\gamma+1}\left(\frac{\partial \upsilon'}{\partial \xi'}\right)_{\tau'}=0. \quad (12)$$

Let us take advantage of key relationship between independent variables in the Eulerian coordinates

$$dr'=(1-\varepsilon)\,dr+\varepsilon u\,dt \quad (13)$$

appearing in the transformation for planar motions ($\nu=1$) [8,9]. Owing to relation (5) the terms with ε collected together in (13) yield the value $\varepsilon dr-\varepsilon u dt=\varepsilon_0\,d\xi$. Then the relation (13) has a form $dr'=dr-\varepsilon_0\,d\xi$, that confirms the physical interpretation for (13), namely, the volume (in planar case ($\nu=1$) the distance) occupied by incompressible phase $\varepsilon_0\,d\xi$ can be eliminated.

The suggestion (13) enable us to assume that the connection between variables r and r' for any symmetry could be as follows:

$$r'^{\nu-1}dr'=r^{\nu-1}dr-\varepsilon_0\xi^{\nu-1}\,d\xi. \quad (14)$$

Thus we satisfy the important condition, namely, that the value dr' is an exact differential. That in turn enable us to rewrite the relationship (14) in integral form:

$$r'^\nu=r^\nu-\varepsilon_0\xi^\nu. \quad (15)$$

The connection between mass velocities follows immediately from (14)

$$r'^{\nu-1}u'=r^{\nu-1}u. \quad (16)$$

Substitution of the relation (15) directly into the condition (10) reduces Eq. (10) to the transformation

$$\xi'^\nu=(1-\varepsilon_0)\xi^\nu. \quad (17)$$

Trying to transform Eq. (11) into (12) we can obtain new equation in which in addition to all terms of the equation (12) we have unfortunately additional term, namely,

$$u'\left(\frac{r'}{r}\right)^{\nu-1}\left(\frac{\partial(r/r')^{\nu-1}}{\partial\tau}\right)_{\xi}. \qquad (18)$$

The additional term (18) vanishes for stationary flows as well as for any flows with planar symmetry, and possibly for self-similar motions.

Consequently, the transformation (8), (9), (15)–(17) between the systems of Eqs. (1)–(4) and (6) is valid at least for stationary flows, i.e. one can state that for cylindrical ($\nu=2$) and spherical ($\nu=3$) symmetries, the stationary motion of the two-phase medium is completely similar to the stationary motion of gas.

4. Self-similar motions with shock waves

The above-mentioned transformation allows one to use its advantage for describing the self-similar problems. Let us apply the method for solving the problem related to the strong explosion stage in a two-phase medium.

Let a finite amount of energy E_0 be instantaneously deposited in an infinitely small volume of a two-phase medium. We restrict ourselves to distances from the explosion source where the wave can be considered as strong, i.e. when one can neglect the initial internal energy of the medium by comparison with E_0. We consider the propagation of the shock wave moving with velocity

$$D=\frac{dr_f}{dt}, \qquad (19)$$

where r_f is a place of the shock wave front, $r_f=r_f(t)$ is a function only of a time. Note that $\xi_f=r_f$.

Let us define new dimensionless variables for equation systems describing the two-phase flow (1), (3), (4)

$$P=\upsilon_0 p/D^2,\quad U=u/D,\quad V=\upsilon/\upsilon_0,\quad \mu=\xi/\xi_f,$$

$$\eta=r/\xi_f,\quad \chi=\xi_f/\tau_0 D,\quad z=\frac{\xi_f}{D^2}\frac{dD}{d\tau}, \qquad (20)$$

as well as gas (6)

$$P'=\upsilon_0' p'/D'^2,\quad U'=u'/D',\quad V'=\upsilon'/\upsilon_0',\quad \mu'=\xi'/\xi_f',$$

$$\eta'=r'/\xi_f',\quad \chi'=\xi_f'/\tau_0 D',\quad z'=\frac{\xi_f'}{D'^2}\frac{dD'}{d\tau'},\quad D'=\frac{d\xi_f'}{d\tau'}. \qquad (21)$$

According to (15) we write

$$r_f'^{\nu}=(1-\varepsilon_0)r_f^{\nu},\quad r_f'^{\nu-1}D'=(1-\varepsilon_0)r_f^{\nu-1}D. \qquad (22)$$

At strong explosion in a two-phase medium the self-similar motion is realized, whereas, the derivatives with respect to χ are equal to zero, and $z=z'=-\nu/2$ (see, for example, [3,6,7,10]). Then we can rewrite the systems of equations for the two-phase medium as follows:

$$\frac{\eta^{\nu-1}}{\mu^{\nu-1}}\frac{d\eta}{d\mu}=V,\quad zU-\mu\frac{dU}{d\mu}+\frac{\eta^{\nu-1}}{\mu^{\nu-1}}\frac{dP}{d\mu}=0,\quad P(V-\varepsilon_0)^{\gamma}\mu^{\nu}=\text{const}, \qquad (23)$$

with boundary conditions at shock wave front

$$U=P=\frac{2(1-\varepsilon_0)}{\gamma+1},\qquad V=\frac{\gamma-1+2\varepsilon_0}{\gamma+1},$$

and for the homogeneous medium (perfect gas) in the form

$$\left(\frac{\eta'}{\mu'}\right)^{\nu-1}\frac{d\eta'}{d\mu'}=V',\quad z'U'-\mu'\frac{dU'}{d\mu'}+\left(\frac{\eta'}{\mu'}\right)^{\nu-1}\frac{dP'}{d\mu'}=0,\quad P'V'^{\gamma}\mu'^{\nu}=\text{const}, \qquad (24)$$

with boundary conditions

$$U'=P'=\frac{2}{\gamma+1},\qquad V'=\frac{\gamma-1}{\gamma+1}.$$

For example, in Appendix B we prove the last equation in (23).

The transformation (8), (9), (15)–(17) is easy reduced to the dimensionless form

$$(1-\varepsilon_0)V'=V-\varepsilon_0,\quad (1-\varepsilon_0)P'=P,\quad (1-\varepsilon_0)\left(\frac{\eta'}{\eta}\right)^{\nu-1}U'=U,$$

$$\eta^{\nu}=(1-\varepsilon_0)\eta'^{\nu}+\varepsilon_0\mu^{\nu},\quad \mu=\mu'. \qquad (25)$$

It turns out that for self-similar motion with shock wave (in contrast to the stationary flow), the transformation between systems (22) and (23) is not succeeded in finding. Anyway for $\nu\neq 1$ there is the difference between system (22) and system appeared from (23) by means of transformation (25). The transformed system contains the additional term $U'\eta'((\eta'/\eta)^{\nu-1})\,d(\eta/\eta')^{\nu-1}/d\eta$.

Using the point explosion as an example, we estimate the error introduced by the additional term. The results of the calculations for strong explosion are demonstrated in Figs. 1–4. We calculate

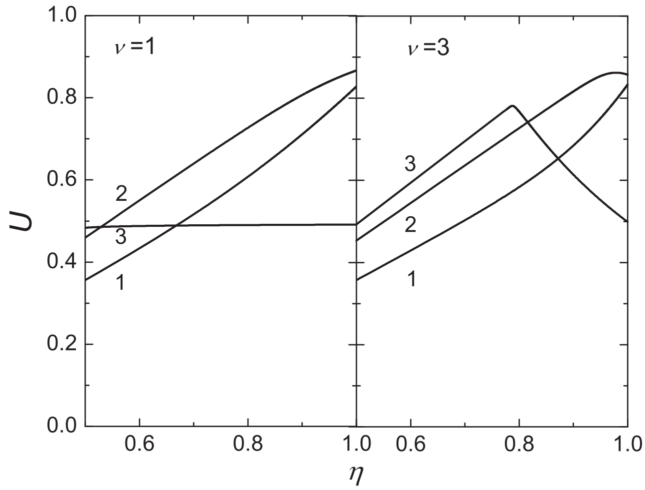

Fig. 1. The profiles of dimensionless velocity U. The solutions calculated by two methods equal each other.

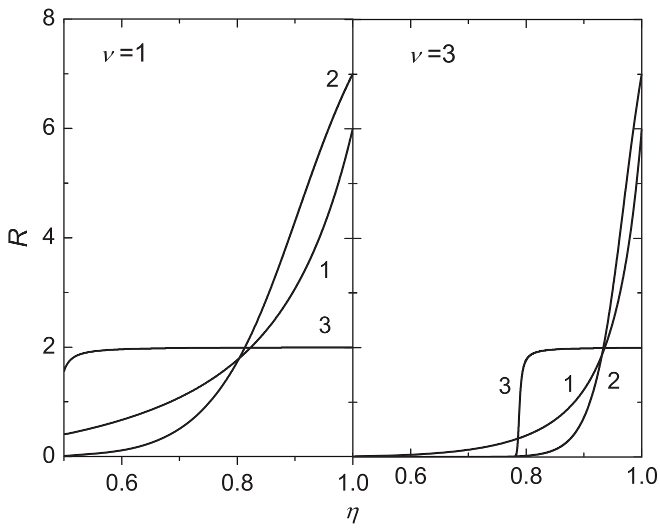

Fig. 2. The profiles of dimensionless density $R=V^{-1}$. The curves calculated by two methods coincide with each other.

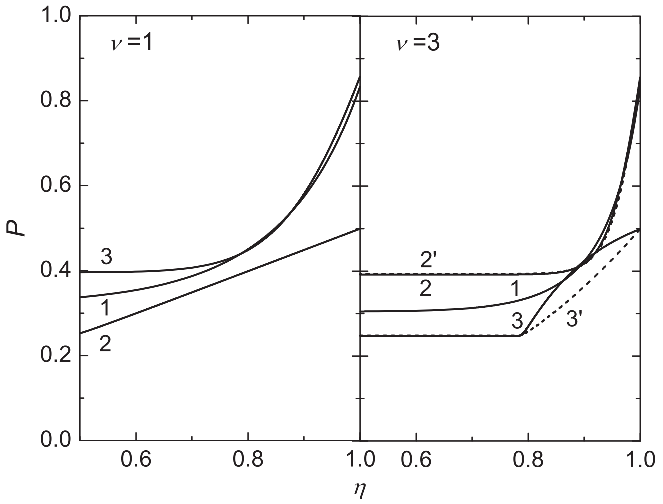

Fig. 3. The profiles of dimensionless pressure P. Curves 2 and 3 are the exact solutions. Curves 2′ and 3′ are the approximate solutions.

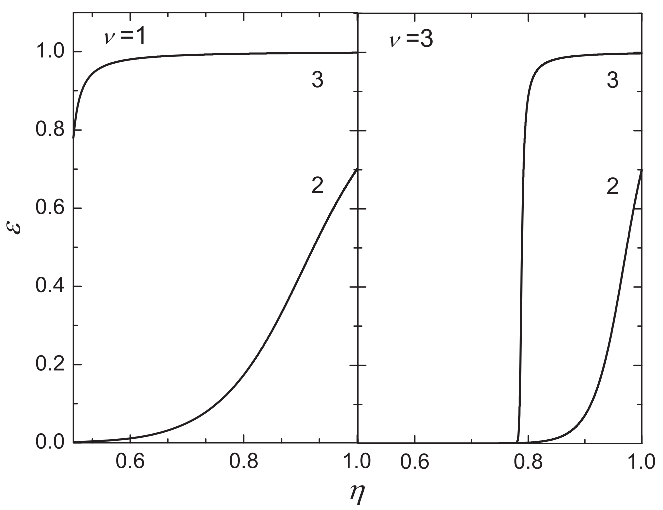

Fig. 4. The distributions of volume portion of incompressible component ε in shock wave.

the dimensionless specific volume V, velocity U and pressure P by two methods. First, the system of Eqs. (23) is directly solved at some particular values ε_0. This is the exact solutions U, V, P. For convenience we use the dimensionless density $R=V^{-1}$ instead of the dimensionless specific volume V. In Figs. 1–3 the variables U, R, P are plotted by curves 1, 2, 3. Second, the variables U, R, P are found by means of the transformation (25) of the solution U', R', P' for perfect gas (24). The obtained then U, R, P are the approximate solution of the equations system (23). The approximate solutions are illustrated by curves 2′, 3′. In Figs. 1–4 the curves 1 relate to gas ($\varepsilon_0=0$, $\gamma=1.4$), curves 2, 2′ and 3, 3′ relate to two-phase media with $\varepsilon_0=0.1$, $\gamma=1.1$ and $\varepsilon_0=0.5$, $\gamma=1.005$, respectively. It is very important that complete agreement is observed for U, R calculated by two methods, therefore, in Figs. 1, 2 the curves 2′, 3′ are not plotted, they are complete coincided with curves 2, 3, respectively. While for the values P at $\nu\neq 1$ (see Fig. 3) the distinction between exact solutions (curves 2, 3) and approximate solutions (curves 2′,3′) are largest. We note that for curves 3, 3′ the initial volume portion is $\varepsilon_0=0.5$, i.e. one-half of an initial volume is occupied by incompressible component. At small $\gamma-1\ll 1$ almost all mass of the condensed phase is accumulated near the front of shock wave (see Fig. 4).

Thus, since for a problem of strong explosion in gas the self-similar solution is known in forms of the analytical dependencies [3,10] and the tabulated data [12], one can obtain with certain accuracy the solution for strong explosion in two-phase medium with incompressible component. Moreover the solution obtained in this manner has analytical dependencies on value of volume portion of incompressible phase ε. Hence, the influence of value ε on two-phase flows can be estimated through analytical dependencies. The example of successful applying the analytical transformation for estimating the velocity of shock wave propagation in two-phase medium is presented in [8,9].

5. Conclusion

We suggest the transformation that enables one to carry over with certain accuracy (for planar symmetry as well as for stationary flows the transformation is exact) the known solutions of gas-dynamic problems to the two-phase media with arbitrary volume portion of incompressible components. This transformation is very important from the viewpoint of the study of the multi-component media. Indeed, this transformation enables one to obtain the solution of many problems for multi-component media with incompressible phase, if there is the similar problem solution for a homogeneous compressible medium. In this case it is not necessary directly to solve the problem for medium with incompressible component, and it is sufficiently only to transform the known solution of the similar problem for a homogeneous medium. Thus, the solutions of many hydrodynamic problems for multi-component media with incompressible phase can be obtained without solving the original system of equations. The scope for the suggested transformation is demonstrated by reference to the strong explosion state in a two-phase medium.

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Appendix A

In this appendix we reduce third equation in (1) to Eq. (11) for adiabatic flow. If the wave propagates in an initial unperturbed medium, then from (4) we have

$$p(\upsilon-\varepsilon_0\upsilon_0)^{\gamma}=p_0(\upsilon_0-\varepsilon_0\upsilon_0)^{\gamma}. \tag{A.1}$$

This relation provides us by derivative

$$\left(\frac{\partial p}{\partial \xi}\right)_{\tau}=-\gamma p_0\frac{\upsilon_0^{\gamma}(1-\varepsilon_0)^{\gamma}}{(\upsilon-\varepsilon_0\upsilon_0)^{\gamma+1}}\left(\frac{\partial \upsilon}{\partial \xi}\right)_{\tau}=-\gamma p_0\left(\frac{\upsilon_0'}{\upsilon'}\right)^{\gamma+1}\frac{1}{\upsilon_0(1-\varepsilon_0)}\left(\frac{\partial \upsilon'}{\partial \xi}\right)_{\tau}. \tag{A.2}$$

Finally, substituting (A.2) in third equation in (1), we obtain the relationship (11).

Similarly one can prove that third equation in (6) is reduced to Eq. (12).

Appendix B

Here we will prove that $P(V-\varepsilon_0)^{\gamma}\mu^{\nu}=\text{const}$. Let us consider the sequence of relations, taking into account (4) and (20),

$$\left(\frac{\partial P(V-\varepsilon_0)^{\gamma}\mu^{\nu}}{\partial \tau}\right)_{\xi}=\left(\frac{\partial \upsilon_0 pD^{-2}(\upsilon-\varepsilon_0\upsilon_0)^{\gamma}\upsilon_0^{-\gamma}(\xi/\xi_f)^{\nu}}{\partial \tau}\right)_{\xi}$$

$$= \upsilon_0^{1-\gamma} D^{-2} (\xi/\xi_f)^\nu \left(\frac{\partial p(\upsilon - \varepsilon_0 \upsilon_0)^\gamma}{\partial \tau} \right)_\xi + \upsilon_0^{1-\gamma} \xi^\nu p(\upsilon - \varepsilon_0 \upsilon_0)^\gamma \left(\frac{\partial D^{-2} \xi_f^{-\nu}}{\partial \tau} \right)_\xi$$

$$= 0 + \upsilon_0^{1-\gamma} \xi^\nu p(\upsilon - \varepsilon_0 \upsilon_0)^\gamma \left(-2 \xi_f^{-\nu} D^{-3} \frac{dD}{d\tau} - \nu \xi_f^{-\nu-1} D^{-2} \frac{d\xi_f}{d\tau} \right)$$

$$= \upsilon_0^{1-\gamma} \xi^\nu p(\upsilon - \varepsilon_0 \upsilon_0)^\gamma (-2 \xi_f^{-\nu-1} D^{-1} z - \nu \xi_f^{-\nu-1} D^{-1}) = 0. \quad \text{(B.1)}$$

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