Diagnostics of the Medium Structure by Long Nonlinear Wave

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Abstract. An asymptotic averaged model for describing the long non-linear wave propagation in structured media is suggested. In the general case, the equations system cannot be reduced to the average hydrodynamic terms. On a microstructural level of a medium, the dynamic behaviour adheres only to the thermodynamic laws, wherein the change of the structure eventually affects the macro wave motion. The important result of this model is that the structure of a medium always increases the non-linear effects under the propagation of long waves, and that non-linearity takes place even for media with the components described by the linear law. This effect forms the theoretical fundamentals of a new method for diagnostics of the properties of medium components by long non-linear waves. The mass contents of components in the medium can be determined by this diagnostic method.

ASYMPTOTIC AVERAGED MODEL

The long wave perturbations have been investigated by using as an example a medium with regular structure. It is supposed that the microstructure elements of the medium ε are large enough that it is possible to submit to the laws of the classic continuum mechanics. The analysis is based on the hydrodynamic approach. This restriction can be imposed for the modeling of non-linear waves in water-saturated soils, bubble media, aerosols, etc. The set of acceptable media could be extended to solid media where the powerful loads are studied in the condition that the strength and plasticity of the material can be neglected. We regard that there are equalities of the stresses as well as of mass velocities on boundaries of neighbouring components. Individual components of the medium are considered to describe by the classical equations of hydrodynamics and state equation (in the Lagrangian mass coordinates)

$$\frac{\partial V}{\partial t}-\frac{\partial u}{\partial m}=0, \qquad \frac{\partial u}{\partial t}+\frac{\partial p}{\partial m}=0, \qquad dp=c^2 d\rho. \tag{1.1}$$

The notations are as generally accepted. Conditions for matching are the equality of the mass velocities and of pressures at boundaries of components.

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We have studied a heterogeneous medium by a method of the asymptotic averaging, which is the combination of a multiscale method with a space averaging method. According to this method, the mass space coordinate m can be split into two independent coordinates: slow one, s, and fast one, ξ. Then $\frac{\partial}{\partial m}=\frac{\partial}{\partial s}+\varepsilon^{-1}\frac{\partial}{\partial \xi}$. The slow coordinate s corresponds to a global variation of the wave field while the fast one ξ traces variations of the field within the structure period. The variables p, u, V are presented as series, for example, $V(m,t)=V^{(0)}(s,\xi,t)+\varepsilon V^{(1)}(s,\xi,t)+\varepsilon^2 V^{(2)}(s,\xi,t)+\dots$.

After integrating Eqs.(1.1) over the structure period, we obtain the averaged system of equations [1]

$$\frac{\partial\langle V^{(0)}\rangle}{\partial t}-\frac{\partial u^{(0)}}{\partial s}=0,\quad \frac{\partial u^{(0)}}{\partial t}+\frac{\partial p^{(0)}}{\partial s}=0,\quad d\langle V^{(0)}\rangle=-\langle (V^{(0)})^2/c^2\rangle dp. \tag{1.2}$$

Here $\langle\cdot\rangle=\int_0^1(\cdot)d\xi$. We are restricted hereafter to zero approximations, and superscript (0) can be omitted. We define the fast Eulerian coordinate ζ as

$$(\partial\zeta/\partial\xi)_t=(\langle V\rangle\rho(\xi))^{-1}. \tag{1.3}$$

The averaged system of Eqs.(1.2) is integro-differential and is not reduced to the averaged variables p, u, $\langle V\rangle$. Indeed, the individual components have the different compressibility. Hence, the change of the averaged specific volume $\langle V\rangle$ differs from the change of specific volume for a homogeneous medium V_{hom}. Consequently, the non-linear waves in a structured medium cannot be modelled by means of a homogeneous medium even for long waves.

On a microstructured level of the medium, the action is statically uniform (waveless). The behaviour of the medium adheres only to the thermodynamic laws. On a macro level, the motion of the medium is described by the wave dynamic laws for averaged variables. Eqs.(1.2) do not change their form if the components are broken and/or change their location in an elementary cell. This means that Eqs.(1.2) describe the motion of any quasi-periodic (statistically heterogeneous) medium. Thus, the asymptotic averaged model describes the long non-linear wave propagation in the structured medium [1].

NON-LINEAR EFFECTS IN STRUCTURED MEDIUM

We have shown [2] that the medium structure always increases the non-linear effects under the propagation of long waves. In [2] we have obtained an evolution equation that takes into account a weak non-linearity (in the Eulerian coordinates x, t)

$$\left(\frac{\sqrt{\langle V^2/c^2\rangle}}{\langle V\rangle}\frac{\partial}{\partial t}-\frac{\partial}{\partial x}\right)p'-\frac{1}{2}\left\langle\frac{V^2}{c^2}\right\rangle^{-1}\frac{d^2\langle V\rangle}{dp^2}p'\frac{\partial p'}{\partial x}=0. \tag{2.1}$$

A coefficient of the non-linearity β, when the sound velocities in the individual components are independent of the pressure, can be presented as

$$\beta=\frac{1}{2}\langle V\rangle\left\langle\frac{V^2}{c^2}\right\rangle^{-3/2}\frac{d^2\langle V\rangle}{dp^2}=\langle V\rangle\left\langle\frac{V^3}{c^4}\right\rangle\left\langle\frac{V^2}{c^2}\right\rangle^{-3/2}. \tag{2.2}$$

For all cases $\beta>0$. Let us consider the ratio of the non-linearity coefficients for heterogeneous and homogeneous media

$$\frac{\beta}{\beta_{\text{hom}}}=\langle V\rangle\left\langle\frac{V^3}{c^4}\right\rangle\left\langle\frac{V^2}{c^2}\right\rangle^{-2}\geq 1. \tag{2.3}$$

So, in a heterogeneous medium β is always greater than β_{hom} in a homogeneous medium. Thus, it is proved that the heterogeneity in a medium structure introduces additional non-linearity. This effect provided the basis for a new method of diagnostics to define the properties of multicomponent media using the propagation of long non-linear waves in such media.

FUNDAMENTALS OF NEW DIAGNOSTIC METHOD

We describe our new diagnostic method for the properties of a medium. The effect of the increase of non-linearity in the heterogeneous medium in comparison with homogeneous medium forms the theoretical fundamentals for the diagnostic method. In this method the properties of medium heterogeneities are defined by long waves of finite amplitudes. We have found [2] that the dependence V/c^2 on fast Eulerian coordinate ζ (see Eq.(1.3)) is defined through the inverse Fourier transform

$$\zeta(V/c^2)=F^{-1}\left[\sum_{n=0}^{\infty}\frac{\langle V(V/c^2)^{n+1}\rangle}{(n+1)!\langle V\rangle}\ \mathrm{i}^n q^n\right](V/c^2). \tag{3.1}$$

Application of the suggested method is connected with the finding of the coefficients $\langle V(Vc^{-2})^n\rangle$ for power series (3.1). These coefficients can be easily calculated, if we know the functional dependence $\langle V\rangle(p)$ or $\langle V/c^2\rangle(p)$. They can be successively defined by the recurrence relation

$$\frac{d\langle V(Vc^{-2})^n\rangle}{dp}=-(n+1)\langle V(Vc^{-2})^{n+1}\rangle \tag{3.2}$$

that follows directly from the state equation (1.1).

Certain difficulties for the application of this method can be connected with the following. The experimental data are always defined with some accuracy, and the application of the formula (3.2) will lead to the increase of the magnitude of error for high-order derivatives. This requires that a limited number of the terms should be used in the series (3.1). We have studied the accuracy of the reconstruction of medium structure in the case when we know only several first terms in the series (3.1).

The partial sum of series (3.1) approximates the desired function $\zeta = \zeta(V/c^2)$ with certain accuracy, namely the diagnosed medium can be approximated by a layer medium [2]. We present as an example the results of the calculation to define the structure of layer media, which can properly approximate the diagnosed medium for the case of $V/c^2 = 0.2 + 0.8(1-\zeta)^2$ (see Fig.1). In order to approximate the diagnosed medium by layer periodic medium, which has N layers within the period, it is necessary to know $2N-1$ values $\langle V(Vc^{-2})^n \rangle$. The distributions of V/c^2 within the period for diagnosed medium (dotted curve) and for approximated media with N components are shown in Fig.1. So, we have illustrated the accuracy of the approximation of the diagnosed medium by the finite series (3.1). Thus, the new method for the diagnostics of the medium characteristics by long non-linear waves is suggested on the basis of the asymptotic averaged model of structured medium.

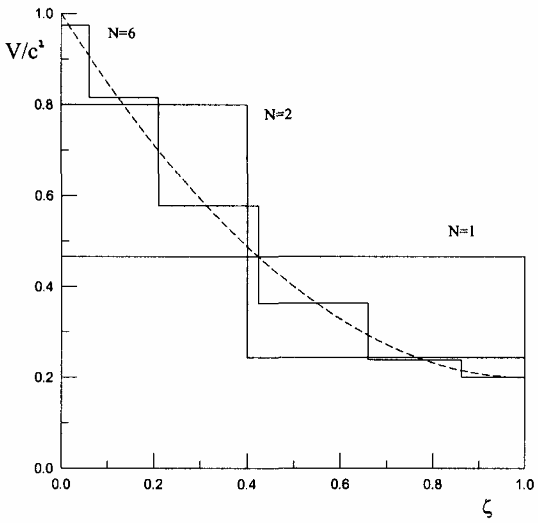

FIGURE 1. Approximation of the diagnosed medium (dotted curve) by N-component media.

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