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NONLINEAR WAVES AS A TECHNICAL TOOL FOR DIAGNOSTICS OF THE MEDIUM STRUCTURE

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Abstract
An asymptotic model (system of averaged equations) for describing the long nonlinear wave propagation in structured media is suggested. In the general case, the equations system cannot be reduced to the average hydrodynamic terms. On a microstructural level of a medium, the dynamic behaviour adheres only to the thermodynamic laws, wherein the change of the structure eventually affects the macro wave motion. This model enables us to obtain the important result, namely, the structure of a medium always increases the nonlinear effects under the propagation of long waves, and that nonlinearity takes place even for media with the components described by the linear law. This effect forms the theoretical fundamentals of a new method for diagnostics of the properties of medium components by long nonlinear waves. The mass contents of components in the medium can be determined by this diagnostic method.

INTRODUCTION

Traditionally, it was considered that in heterogeneous media with wavelength appreciably exceeding the size of the structural heterogeneities, the perturbations propagate in the same way as in homogeneous media. Recent experiments have shown that it is necessary to take into account the inner structure of a medium under propagation of the nonlinear wave perturbations. The information contained in the wave field evolution can be used as a tool when we want to establish the properties of the medium itself as well as to study the effects on various objects.

Models with the different degrees of complexity have been used to describe the wave processes in heterogeneous media. In the framework of continuum mechanics the known idealization of a real medium as homogeneous has enabled considerable success for describing wave processes. The continuum models are commonly applied to the mixtures in which the dispersive dissipative properties take into account the interactions between the components. On this level the media are modeled in the framework of a homogeneous elastic, viscous elastic and elastic plastic media. In this case the features of a medium's structure are taken into account indirectly through the kinetic parameters (relaxation time, the viscous coefficients etc.).

In all the above models, the formalism of continuum mechanics which was used is based on the principle of local action as well as on the generalization of the mechanics laws relating the point mass to the continuum.

When going from integral equations to differential balance equations, the existence of a differentially small microvolume dv is assumed. On the one hand, this volume is so small that the mechanics laws of the point mass have been expanded to the whole microvolume and, on the other hand, the volume contains so many structural elements of the medium that, in this sense, it can be regarded as macroscopic in spite of its smallness as compared to the entire volume occupied by the medium. So, the transition to the differential balance equations is based on the assumption of smallness of microstructural scales ε in comparison to the characteristic macroscopic scale of the flow λ, and transition should be made to the limit $\varepsilon / \lambda \to 0$. Contraction of the volume dv to the point, in the general case, is correct for the continuous functions. This means that all points inside differentially small volume are equivalent. Hence if the mixture is considered, the equivalence of the points means that the field characteristics averaged over dv should be used. Consequently, it is assumed that the equations of motion can be written using the average terms such as density, mass velocity and pressure, which are ascribed to the each component separately. We note that in these models the sizes of components are not explicitly included.

The application of models of a homogeneous medium to the description of the dynamic wave processes in a structured natural medium encounters certain principle difficulties. We will take into account the structure of a medium on a macrolevel. We abandon the assumption that the differentially small volume dv contains all components of medium, although we will consider the long wave approach when wavelength λ is much larger than the characteristic length of structure of a medium ε. We consider a structured medium (Fig.1) in which separated components are considered as a homogeneous medium (the differentially small volume dv is much smaller than the characteristic size of a particular component).

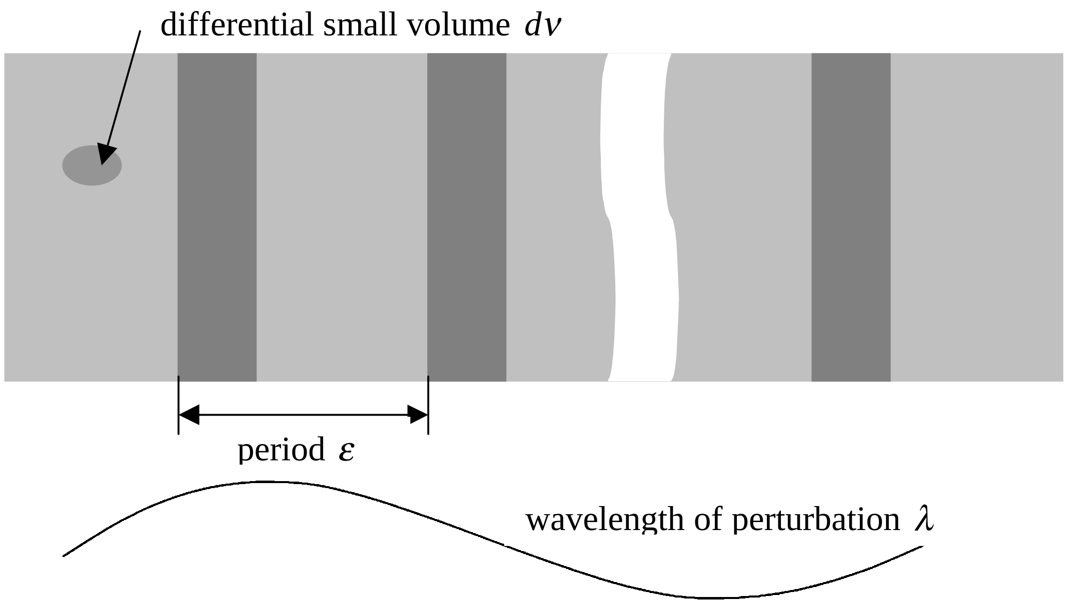


Fig. 1. Model medium with two homogeneous components in period

The simplest case of heterogeneous media for which the effect of structure can be analyzed are media with a regular structure. Features of the propagation of long wave perturbations has been investigated by using as an example a periodic medium under conditions of an equality of stresses and mass velocities on boundaries of neighboring components. It is supposed that the microstructure elements of medium ε (see Fig.1) are large enough that it is possible to submit to the laws of classical continuum mechanics. Also we suppose that a medium is barothropic. We consider that the properties of the medium, such as density, sound velocity and relaxation time vary in a periodic manner (although this assumption is unessential in the final result). We have used the hydrodynamic approach and considered the media without tangential stresses. This restriction is imposed for the modelling of powerful loads as well as nonlinear waves in water-saturated soils, bubble media, etc.

ASYMPTOTIC AVERAGED MODEL FOR STRUCTURED MEDIUM

In the Lagrangian coordinate system (l,t) the equations of one-dimension motion for each element of the regular structure medium have the form

$$\frac{\partial V}{\partial t}-\frac{\partial u}{\partial m}=0, \qquad \frac{\partial u}{\partial t}+\frac{\partial p}{\partial m}=0, \qquad dp=c^2 d\rho . \tag{1}$$

The notations are as generally accepted. Conditions for matching are the equality of the mass velocities and of pressures at boundaries of components. We have studied a heterogeneous medium by a method of the asymptotic averaging, which is the combination of a multiscale method with a space averaging method. According to this method, the mass space coordinate m can be split into two independent coordinates: slow one, s, and fast one, ξ. Then $\frac{\partial}{\partial m}=\frac{\partial}{\partial s}+\varepsilon^{-1}\frac{\partial}{\partial \xi}$. The slow coordinate s corresponds to a global variation of the wave field while the fast one ξ traces variations of the field within the structure period. The variables p, u, V are presented as series, for example, $V(m,t)=V^{(0)}(s,\xi,t)+\varepsilon V^{(1)}(s,\xi,t)+\varepsilon^2 V^{(2)}(s,\xi,t)+\ldots$. After integrating Eqs.(1) over the structure period, we obtain the averaged system of equations [1]

$$\frac{\partial\left\langle V^{(0)}\right\rangle}{\partial t}-\frac{\partial u^{(0)}}{\partial s}=0, \qquad \frac{\partial u^{(0)}}{\partial t}+\frac{\partial p^{(0)}}{\partial s}=0,$$
$$d\left\langle V^{(0)}\right\rangle=-\left\langle (V^{(0)})^2/c^2\right\rangle dp. \tag{2}$$

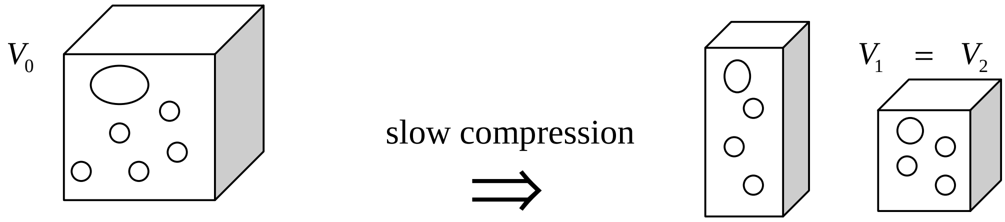
Here $\langle\cdot\rangle=\int_0^1(\cdot)d\xi$. We are restricted hereafter to zero approximations, and superscript (0) can be omitted. We define the fast Eulerian coordinate ζ as

$$(\partial\zeta/\partial\xi)_t=(\langle V\rangle\rho(\xi))^{-1}. \tag{3}$$

The averaged system of Eqs.(2) is integro-differential and is not reduced to the averaged variables p, u, $\langle V\rangle$. Indeed, the individual components have the different compressibility. Hence, the change of the averaged specific volume $\langle V\rangle$ differs from the change of specific volume for a homogeneous medium V_{hom}. Consequently, the nonlinear waves in a structured medium cannot be modelled by means of a homogeneous medium even for long waves.

On a microstructured level of the medium, the action is statically uniform (waveless). The behaviour of the medium adheres only to the thermodynamic laws (see Fig.2).

On a macro level, the motion of the medium is described by the wave dynamic laws for averaged variables. The equations (2) do not change their form if the components are broken and/or change their location in an elementary cell. This means that Eqs.(2) describe the motion of any quasi-periodic (statistically heterogeneous) medium. Thus, the asymptotic averaged model describes the long nonlinear wave propagation in the structured medium [1].



Multicomponent medium

State of medium is determined by termodynamic laws

Fig.2. Motion of microvolum is as whole

NONLINEAR EFFECTS IN STRUCTURED MEDIUM

We have shown [2] that the medium structure always increases the nonlinear effects under the propagation of long waves. In [2] we have obtained an evolution equation that takes into account a weak nonlinearity (in the Eulerian coordinates x, t)

$$\left(\frac{\sqrt{\langle V^2/c^2 \rangle}}{\langle V \rangle} \frac{\partial}{\partial t} - \frac{\partial}{\partial x} \right) p' - \tfrac{1}{2} \left\langle \frac{V^2}{c^2} \right\rangle^{-1} \frac{d^2 \langle V \rangle}{dp^2} p' \frac{\partial p'}{\partial x} = 0 . \tag{4}$$

A coefficient of the nonlinearity β, when the sound velocities in the individual components are independent of the pressure, can be presented as

$$\beta = \tfrac{1}{2} \langle V \rangle \left\langle \frac{V^2}{c^2} \right\rangle^{-3/2} \frac{d^2 \langle V \rangle}{dp^2} = \langle V \rangle \left\langle \frac{V^3}{c^4} \right\rangle \left\langle \frac{V^2}{c^2} \right\rangle^{-3/2} . \tag{5}$$

For all cases $\beta > 0$. Let us consider the ratio of the nonlinearity coefficients for heterogeneous and homogeneous media

$$\frac{\beta}{\beta_{\text{hom}}} = \langle V \rangle \left\langle \frac{V^3}{c^4} \right\rangle \left\langle \frac{V^2}{c^2} \right\rangle^{-2} \geq 1 . \tag{6}$$

So, in a heterogeneous medium β is always greater than β_{hom} in a homogeneous medium. Thus, it is proved that the heterogeneity in a medium structure introduces additional nonlinearity. This effect provided the basis for a new method of diagnostics to define the properties of multicomponent media using the propagation of long nonlinear waves in such media.

FUNDAMENTALS OF NEW DIAGNOSTIC METHOD

We describe our new diagnostic method for the properties of a medium. The effect of the increase of nonlinearity in the heterogeneous medium in comparison with homogeneous medium forms the theoretical fundamentals for the diagnostic method. In this method the properties of medium heterogeneities are defined by long waves of finite amplitudes. We have found [2] that the dependence V/c^2 on fast Eulerian coordinate ζ (see Eq.(3)) is defined through the inverse Fourier transform

$$\zeta(V/c^2) = F^{-1}\left[\sum_{n=0}^{\infty} \frac{\left\langle V(V/c^2)^{n+1}\right\rangle}{(n+1)!\langle V\rangle} \mathrm{i}^n q^n\right](V/c^2). \tag{7}$$

Application of the suggested method is connected with the finding of the coefficients $\left\langle V(Vc^{-2})^n\right\rangle$ for power series (7). These coefficients can be easily calculated, if we know the functional dependence $\langle V\rangle(p)$ or $\left\langle V/c^2\right\rangle(p)$. They can be successively defined by the recurrence relation

$$\frac{d\left\langle V(Vc^{-2})^n\right\rangle}{dp} = -(n+1)\left\langle V(Vc^{-2})^{n+1}\right\rangle \tag{8}$$

that follows directly from the state equation (1).

Certain difficulties for the application of this method can be connected with the following. The experimental data are always defined with some accuracy, and the application of the formula (8) will lead to the increase of the magnitude of error for high-order derivatives. This requires that a limited number of the terms should be used in the series (7). We have studied the accuracy of the reconstruction of medium structure in the case when we know only several first terms in the series (7).

The partial sum of series (7) approximates the desired function $\zeta = \zeta(V/c^2)$ with certain accuracy, namely the diagnosed medium can be approximated by a layer medium [2]. We present as an example the results of the calculation to define the structure of layer media, which can properly approximate the diagnosed medium for the case of $V/c^2 = 0.2 + 0.8(1-\zeta)^2$ (see Fig.3). In order to approximate the diagnosed medium by layer periodic medium, which has N layers within the period, it is necessary to know $2N-1$ values $\left\langle V(Vc^{-2})^n \right\rangle$. The distributions of V/c^2 within the period for diagnosed medium (dotted curve) and for approximated media with N components are shown in Fig.3. So, we have illustrated the accuracy of the approximation of the diagnosed medium by the finite series (7). Thus, the new method for the diagnostics of the medium characteristics by long nonlinear waves is suggested on the basis of the asymptotic averaged model of structured medium.

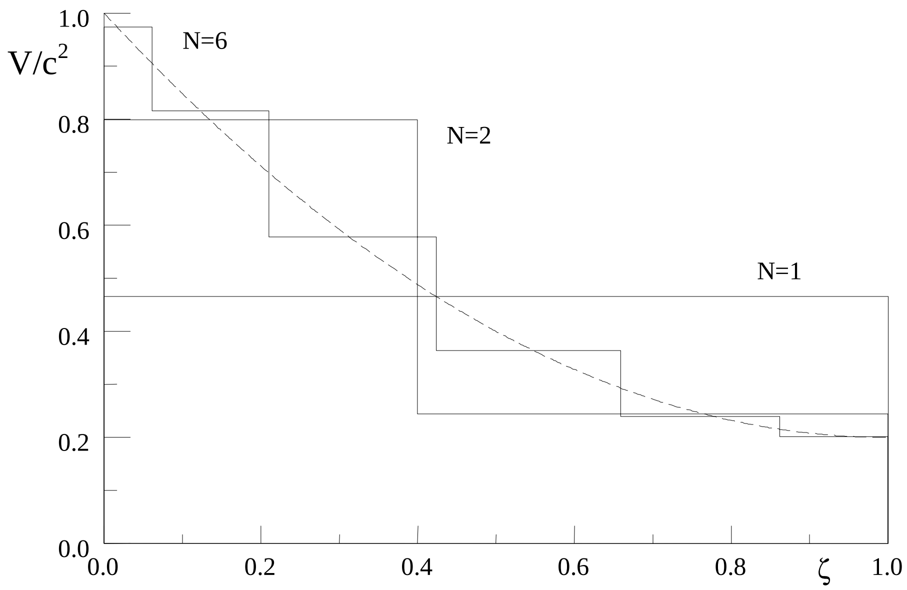


Fig. 3. Approximation of the diagnosed medium (dotted curve) by N-component media.

CONCLUSIONS

The averaged systems of hydrodynamic equations for describing the long wavelength motions in nonhomogeneous media have been discussed. These systems are not expressed in the average hydrodynamic terms and, consequently, the dynamic behaviour of a medium cannot be modeled by means of a homogeneous medium even for long waves, if they are nonlinear. The structure of the medium affects the nonlinear wave propagation. The heterogeneity in a medium structure always introduces additional nonlinearity in comparison with homogeneous medium. This effect enables us to form the theoretical fundamentals of the new diagnostic method to define the characteristics of a heterogeneous medium using the long waves of finite amplitudes (inverse problem). The mass contents of the particular components can be denoted by this diagnostic method.

ACKNOWLEDGEMENT

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