

EFFECT OF THERMAL RELAXATION OF ATTENUATION OF SHOCK WAVES
IN TWO-PHASE MEDIUM

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Molecular relaxation following an explosion in a gaseous or liquid medium proceeds so fast that the perturbation front can be regarded as a discontinuity surface (shock wave). This makes it correct to use the self-adjoint theory of point explosion [3, 6] for describing the evolution of a shock wave during the high-intensity stage of the explosion process also at distances where the shape of the energy source does not play a significant role. In this case the pressure and velocity field building up after an explosion is uniquely determined by the energy of explosion and the thermophysical properties of the gas surrounding the energy source. After explosion in a two-phase medium only the gaseous component reaches equilibrium immediately, owing to the inertia of condensate particles at the base of the wavefront. The hydrodynamic jump is followed by a wide relaxation zone, the equalization time for the parameters of both phases being one order of magnitude longer than the relaxation time for the gas [2, 8]. Consequently, formation of a shock wave after explosion in a two-phase medium cannot any more be regarded as occurring within an infinitesimally short time and for determining the parameters of such a shock wave one must take into account the relaxational character of the wave process [2].

For the purpose of analyzing the effect of relaxation processes on the attenuation of strong shock waves, we will consider a homogeneous two-phase medium consisting of a gaseous phase and a condensate phase. Let an explosion occur in this medium and, as a result, an energy E producing a shock wave be released instantaneously within an infinitesimally small volume. We will make the following assumptions:

1. The density and the specific heat of the condensate phase remain constant.
2. The volume fraction and the partial pressure of the condensate phase are low so that they can be disregarded.
3. The gas obeys the ideal gas law with constant c_p and γ .
4. The gaseous phase and the condensate phase flow at the same velocity.
5. The energy of the mixture is an additive quantity.
6. The time for thermal relaxation between gas and condensate is constant.
7. Transfer of perturbations from the relaxation zone to the front of the shock wave occurs instantaneously.

For describing the motion of the two-phase mixture we let ρ_g , p , c_p , T , ϵ_g denote respectively the density, the pressure, the constant-pressure specific heat, the temperature, and the energy per unit mass of the gaseous phase, σ , c , τ , ϵ_c denote respectively the mass concentration, the specific heat, the temperature, and the energy per unit mass of the condensate phase, r , u , v , denote respectively the distance from the center of symmetry, the mass rate of flow, and the dimensionality of symmetry, and let subscript 0 refer to parameters of gasdynamic discontinuity.

On the basis of the assumptions made here, the fundamental system of equations for the mixture will be put in the form

$$\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} + \rho \left(\frac{\partial u}{\partial r} + \frac{v-1}{r} u \right) = 0; \quad \rho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} \right) + \frac{\partial p}{\partial r} = 0; \quad (1)$$

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$$\frac{\partial}{\partial t} \rho \varepsilon + \frac{\partial}{\partial r} \rho u \varepsilon + p \left(\frac{\partial u}{\partial r} + \frac{v-1}{r} u \right) = 0; \quad \rho \varepsilon = \rho_g \varepsilon_g + \sigma \varepsilon_c; \quad \rho = \rho_g + \sigma. \quad (2)$$

Inasmuch as the condensate phase does not contribute to the pressure, the well known approach to the problem [2, 7] can be used for reducing Eq. (2) to one analogous in form to the equation of state for a gas

$$\rho \varepsilon - \sigma \varepsilon_{c0} = \frac{p}{\Gamma - 1}; \quad (3)$$

$$\Gamma = \gamma \frac{1 + \omega \frac{\tau - T_0}{T}}{1 + \gamma \omega \frac{\tau - T_0}{T}}; \quad \omega = \frac{c}{c_p} \cdot \frac{\sigma}{\rho_g}. \quad (4)$$

With the aid of the equation of state for a two-phase medium in form (3), one can write the equation of a shock adiabat for a two-phase medium as

$$\frac{\rho_f}{\rho_0} = \frac{\frac{\Gamma + 1}{\Gamma - 1} \frac{p}{p_0} + 1}{\frac{\gamma + 1}{\gamma - 1} + \frac{p}{p_0}}. \quad (5)$$

During the high-intensity stage of a shock wave (when $p \gg p_0$) we have, according to Eq. (5), parameters at the wavefront which obey the conventional relations

$$u_f = \frac{2}{\Gamma + 1} D; \quad \rho_f = \frac{\Gamma + 1}{\Gamma - 1} \rho_0; \quad p_f = \frac{2}{\Gamma + 1} \rho_0 D^2, \quad (6)$$

with D denoting the velocity of the shock front. At the center of symmetry $r = 0$ must be $u = 0$.

The foregoing equations and boundary conditions will be supplemented with the integral relation

$$\left[2(v-1)\pi + \frac{1}{2}(v-2)(v-3) \right] \int_0^{r_\phi} \left(\frac{p}{\Gamma - 1} + \rho \frac{u^2}{2} \right) r^{v-1} dr = E. \quad (7)$$

The mathematical form of Eqs. (1)-(3), (6), (7) is analogous to the mathematical form of the equations for a strong point explosion in a gas [6]. This analogy permits us to use existing methods [3, 6, 9] for solving our equations.

Assuming that $T \gg T_0$ behind the front of a strong shock wave, it is easy to demonstrate with the aid of Eq. (3) that $T = \tau$ and $\Gamma = \Gamma_0$ during thermodynamic equilibrium ($\Gamma_0 = \gamma \frac{1 + \omega}{1 + j\omega}$ denoting the exponent of the Poisson adiabat for a two-phase mixture [2, 5, 7]). Experimental studies [8, 11] have demonstrated the correctness of using Γ_0 for a description of equilibrium parameters of shock waves in two-phase media.

On the other hand, during "frozen" relaxation processes $\tau = T_0$ and, according to relation (4), $\Gamma = \gamma$. Therefore, Γ is a function of relaxation parameters behind the front of the shock wave.

For specificity, we will henceforth consider a two-phase medium with $\Gamma_0 \approx 1$. In two-phase media with a foamy structure, for instance, $\Gamma_0 \approx 1$ over the entire $\sigma = 2-50 \text{ kg/m}^3$ range.

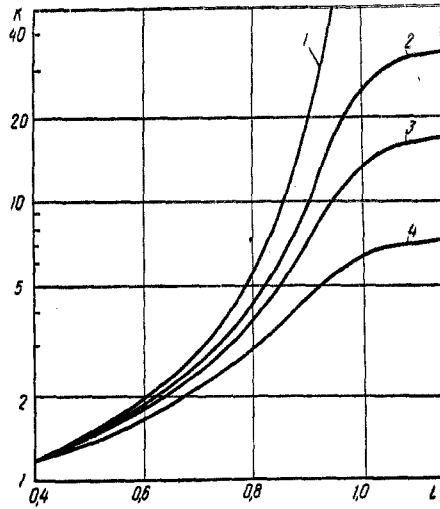


Fig. 1

For solving the system of Eqs. (1)-(3), (6), (7) and determining the effect of relaxation processes on the parameters of a shock wave in a two-phase medium one must specify the law according to which Γ varies behind the wavefront. Let us stipulate that Γ varies exponentially from γ to its equilibrium value Γ_0 as a function of the time f during which a microvolume of the two-phase mixture finds itself within the shock wave

$$\Gamma = \Gamma_0 + (\gamma - \Gamma_0) \exp(-f/t_0). \quad (8)$$

Then at the wavefront $\Gamma = \gamma$ at $r = r_f$. Here t_0 denotes the thermal relaxation time for a two-phase mixture.

For the initial stage of explosion ($t \ll t_0$), when $\Gamma = \gamma$, the system of Eqs. (1)-(3), (6), (7) has a self-adjoint solution [3, 6] at every r . In this solution in $\lambda = r/r_f$, t coordinates the dependent variables, nondimensionalized respectively as

$$V = u/D; \quad P = p/p_0 D^2; \quad \kappa = \rho/\rho_0, \quad (9)$$

remain constant in time.

The time f during which a microvolume finds itself within the shock wave, and the self-adjoint solution is valid, satisfies the differential equation

$$\frac{\partial f}{\partial t} = 1 - \frac{2}{\nu + 2} \cdot \frac{(V - \lambda)}{t} \cdot \frac{\partial f}{\partial \lambda} \quad (10)$$

with the initial and boundary conditions $f = 0$ at $t = 0$ and at $\lambda = 1$.

It is well known [3, 6] that the velocity at the center of symmetry varies according to the law $V = \lambda/\gamma$. We will start with the assumption that this law remains valid over the entire range of λ . Then the solution of Eq. (10) will be

$$f = t(1 - \lambda^a); \quad a = \frac{\gamma(\nu + 2)}{2(\gamma - 1)}. \quad (11)$$

For further analysis we assume that transfer of perturbation from the relaxation zone to the front of a shock wave occurs at a much faster rate than the rate of change of Γ . It follows from this assumption that the terms in Eqs. (1) which include partial derivatives with respect to time are each much smaller when written in variables (9). If it were possible to introduce some quantity Γ' constant over the volume and characterizing the given process, therefore, then the values of variables V , P , and κ would appear in a series of self-adjoint solutions differing only in the value of Γ' .

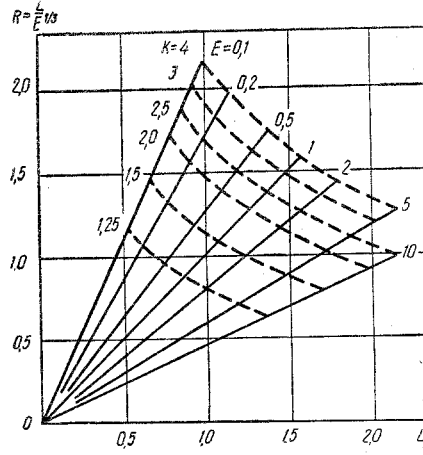


Fig. 2

In order to achieve this, we average Γ over the volume in the first approximation on the basis of the equation for the total energy with the assumption that $P = \text{const}$

$$\int_0^1 \frac{1}{\Gamma-1} \lambda^{\nu-1} d\lambda = \frac{1}{\Gamma'-1} \int_0^1 \lambda^{\nu-1} d\lambda. \quad (12)$$

We transform the left-hand side, using the variables (9) and the properties of the incomplete gamma function $\gamma'(\nu/a, t/t_0)$ [10]

$$\begin{aligned} \int_0^1 \frac{\lambda^{\nu-1}}{\Gamma-1} d\lambda &= (\gamma - \Gamma_0) \int_0^1 \exp[t/t_0 (1 - \lambda^a)] \lambda^{\nu-1} d\lambda = (\gamma - \Gamma_0) \frac{\exp(t/t_0)}{a (t/t_0)^{\nu/a}} \gamma' \left(\frac{\nu}{a}, \frac{t}{t_0} \right) \\ &= \frac{\gamma - \Gamma_0}{\nu} \sum_{l=0}^{\infty} \frac{(t/t_0 \nu/a)^l}{(1 + \nu/a)(1 + 2\nu/a) \dots (1 + l\nu/a)}. \end{aligned} \quad (13)$$

This series can be approximated with $\exp\left(t/t_0 \cdot \frac{1}{1 + \nu/a}\right)$ when $t \ll t_0$ and with e^{t/t_0} when $t \gg t_0$.

One can thus introduce a new variable, constant over the volume, which characterizes the relaxation process

$$\Gamma' = \Gamma_0 + (\gamma - \Gamma_0) \exp(-t/nt_0), \quad (14)$$

where $1 \leq n \leq 1 + \frac{2\nu(\gamma-1)}{\gamma(\nu+2)}$, so that, for instance, $1 \leq n \leq 1.34$ for $\gamma = 1.4$ and $\nu = 3$.

In subsequent calculations n will be regarded as a constant, inasmuch as it varies only slightly in time.

We differentiate Eqs. (7) with respect to time, using the variable (14), which yields for the velocity of the shock front $D = dr_f/dt$ [9]

$$\frac{r_f}{D^2} \cdot \frac{dD}{dt} = -\frac{\nu}{2} - \frac{1}{2nt_0} \cdot \frac{r_f}{D} \cdot \frac{\Gamma' - \Gamma_0}{\Gamma' - 1}, \quad (15)$$

with $r_f = 0$ at $t = 0$.

The solution to this equation is

$$r_f = A \left\{ 2nt_0 \left[1 - x + \frac{b}{2} \ln \left(\frac{1-b}{1+b} \cdot \frac{1+b/x}{1-b/x} \right) \right] \right\}^{2/\nu+2}, \quad (16)$$

with the notation $x = \sqrt{\frac{\Gamma' - 1}{\gamma - 1}}$; and $b = \sqrt{\frac{\Gamma_0 - 1}{\gamma - 1}}$.

The constant A is evaluated from the condition that at $t \rightarrow 0$ the solution must become self-adjoint, this constant being related to the explosion energy E and to the initial density of the medium ρ_0 through the coefficient α [2, 3]:

$$A = \left(\frac{E}{\alpha \rho_0} \right)^{1/\nu+2}. \quad (17)$$

At the initial instant of time, when $\frac{\Gamma_0 - 1}{\Gamma - 1} \ll 1$, the solution does not depend on Γ_0 and is

$$r_f = A [2nt_0 (1 - \exp(-t/2nt_0))]^{2/\nu+2}. \quad (18)$$

According to definition, the pressure attenuation coefficient K for a two-phase medium is the ratio of pressure of the shock wave in the medium without thermal relaxation to pressure at the same distance in a medium of the same density with heat transfer between phases. According to relations (6), the attenuation coefficient can be expressed as

$$K = \frac{\Gamma' + 1}{\gamma + 1} \left(\frac{D_A}{D} \right)^2, \quad (19)$$

with D and D_A denoting the velocity of shock fronts respectively with and without heat transfer occurring.

It can be demonstrated that K depends on Γ'

$$K = \frac{(\gamma - 1)(\Gamma_0 + 1)}{(\Gamma' - 1)(\gamma + 1)}. \quad (20)$$

The attenuation of shock waves will, obviously, be the maximum attainable when the relaxation process run their full course and then $K = \frac{(\gamma - 1)(\Gamma_0 + 1)}{(\Gamma_0 - 1)(\gamma + 1)}$, so that, for instance, $K = 33.4$ when $\gamma = 1.4$ and $\Gamma_0 = 1.01$ respectively.

At the initial instant of time, when heating of the condensate phase is still insignificant ($\Gamma_0 - 1 \ll \Gamma' - 1$), the pressure attenuation coefficient depends on the time in which the shock wave reaches a certain distance in the relaxing medium and does not depend on Γ_0 , i.e., on the condensate concentration:

$$K = \exp\left(\frac{t}{nt_0}\right). \quad (21)$$

Upon examining the dependence of the initial value of the pressure attenuation coefficient on the relative radius $R = L/E^{1/\nu}$, the explosion energy E, and the initial density ρ_0 , we arrive at the relation

$$K = \left(1 - \frac{\sqrt{\alpha}}{2nt_0} R^{\frac{\nu+2}{2}} E^{1/\nu} \rho_0^{1/2} \right)^{-2}. \quad (22)$$

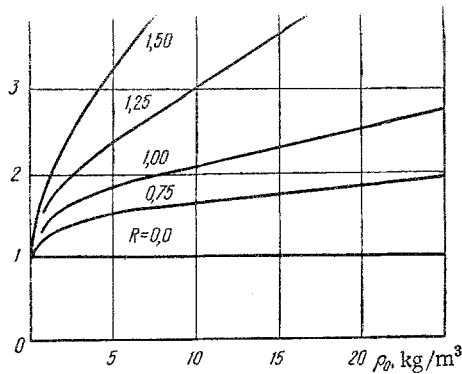


Fig. 3

In the absence of heat transfer between phases the solution to the system of Eqs. (1)-(3), (6), (7) becomes self-adjoint, with the pressure attenuation coefficient (20) equal to unity and depending neither on the explosion energy nor on the relative radius. The same result is obtained from relation (22) at the limit $t_0 \rightarrow \infty$.

Expressions (21) and (22) for K are also valid in the case of foams with high concentration of the condensate phase, when Γ_0 approaches unity.

Curve 1 in Fig. 1 depicts the dependence of the calculated pressure attenuation coefficient on the dimensionless distance $L = r_f/A(2nt_0)^{2/\nu+2}$ in media with $\gamma = 1.4$, $\nu = 3$, $c/c_p = 4$, $\rho_g = 1.3 \text{ kg/m}^3$, and a mass concentration of the condensate phase $\sigma \rightarrow \infty$ (which corresponds to $\rho_c \rightarrow \infty$ under the constraints imposed by smallness of the volume fraction of the condensate phase). This curve approaches asymptotically the limit $L = 1$. Accordingly, a wave can reach only a certain distance which is determined on the one hand by the thermo-physical properties of the medium and the explosion energy but on the other hand also by the kinetics of relaxational interaction between the phases. Curves 2, 3, and 4 in Fig. 1 depict the dependence of calculated K on L for a medium with mass concentrations of the condensate phase $\sigma = 10, 5, \text{ and } 2 \text{ kg/m}^3$ respectively. It is noteworthy that these curves reach the saturation level within a narrow range of the dimensionless distance for all the three different concentrations. The dimensionless group L can, therefore, be regarded as a similitude criterion for determining the maximum attenuation of shock waves in media with thermal relaxation.

The graph in Fig. 2 depicts the interrelation between four quantities: distance L to the shock front, relative radius $R = L/E^{1/\nu}$, relative explosion energy E , and pressure attenuation coefficient K . One can trace here the variation of any three of them as well as of them together. According to this graph, the pressure attenuation coefficient increases with increasing distance at constant explosion energy as well as with increasing explosion energy at a fixed relative radius R .

The curves in Fig. 3 depict the dependence of coefficient K on the density ρ_0 at various relative radii R . According to these curves, an increase of the density of the medium causes the pressure attenuation coefficient to increase only slightly near the explosion center but quite appreciably at larger radii R . This trend is attributable to expenditure of more energy on heating the condensate in a denser medium as the velocity of the propagating wave decreases and to a longer time required by the wave in a denser medium to reach a certain point in space. It is noteworthy that in the absence of heat transfer between the phases $\Gamma = \gamma$, according to relation (4), and relations (16), (17), (19) yield a pressure attenuation coefficient which does not depend on the density of the medium.

The graph in Fig. 4 depicts experimental data on the dependence of the pressure drop at the wavefront and of the velocity of the shock wave generated by detonation of an explosive substance in foam with a mass concentration of liquid $\sigma = 15 \text{ kg/m}^3$ (dash line) and in air (solid line) on the referred distance $R^* = r_f Q^{1/3}$ [2], with r_f (m) denoting the distance from the explosion center and Q (kg) denoting the mass of the hexogen charge. The experiments were performed with hexogen charges weighing 1-3 kg. A comparison of the experimental velocity-pressure curves indicates the relation between maximum pressure and wave velocity, within the given ranges of condensate concentration and explosion energy, can be described by the

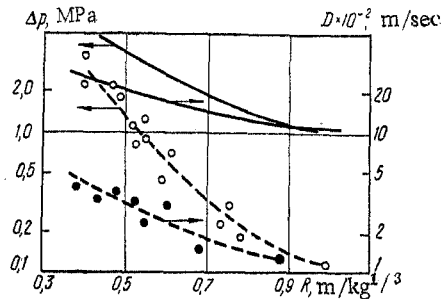


Fig. 4

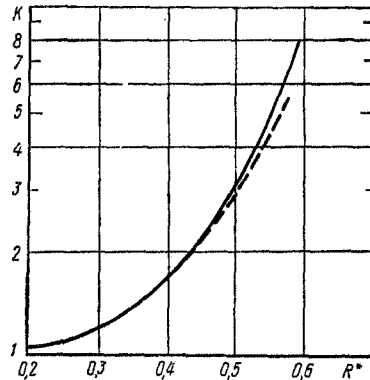


Fig. 5

third of expressions (6). Such a relation is possible only under condition of kinematic equilibrium between the gaseous phase and the condensate phase.

Since the pressure attenuation coefficient under conditions of kinematic phase equilibrium depends only on the heat transfer between phases, it becomes in such a case possible to deduce the trend of thermal relaxation in a two-phase medium, without distortion, from experimental data on attenuation of shock waves. In order to establish the model kinetics here, it is appropriate to compare the experimental curves of shock wave attenuation in foams and in a medium describable by the equation of state for a gas but not endowed with relaxation properties. As such a medium can, specifically, serve a gaseous one and available data on explosion in air [1] can be used for this purpose.

From these experimental data one calculates the pressure attenuation coefficient as the ratio of pressure in air to pressure in foam at a fixed value of the dimensionless distance. It must be taken into consideration that the thus calculated pressure attenuation coefficient K within the zone for which the energy source cannot be regarded as a point source will be lower than that calculated theoretically. This is so because a shock wave begins to "disregard" the nonideality of the energy source at nearer distances in a denser medium, in this case in foam, than in air [4]. The pressure at the front of a shock wave attributable to nonideality of the energy source within the near zone of shock wave buildup, for instance, should be higher in foam than in air.

For predicting the attenuation of shock waves in foam on the basis of the proposed model one needs to have an experimental point as reference, inasmuch as the parameter t_0 is involved. In this study $R^* = 0.4 \text{ m/kg}^{1/3}$ was used as the reference point for calculations and t_0 was found to be 180 μsec .

The graph in Fig. 5 depicts the dependence of the pressure attenuation coefficient on the dimensionless radius R^* , according to experiment (dash line) and according to theory (solid line). A comparison of the two curves indicates that it is possible to describe the trend of shock wave attenuation in a two-phase medium with $\Gamma_0 \approx 1$ and to evaluate the parameters of a shock wave during the high-intensity stage of the explosion within the scope of the proposed model. Disregarding the counterpressure will, as was to be expected, result in an overestimation of the pressure attenuation coefficient at $R^* > 0.6 \text{ m/kg}^{1/3}$.

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