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ESTIMATION OF ROCK FAILURE ZONE UNDER CONFINED EXPLOSION

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The model of rock-mass shattering is constructed with due regard for the wave load attenuation governed by the geometrical divergence of wave and irreversible losses in rock. The geometrical similarity of failure zone is proved for explosions of different energy. The analytical relations connecting the blast wave characteristics and failure parameters are established.

Blast loads, failure zone, rock mass

INTRODUCTION

In many problems of blasting, the formation of cracks in a rock mass under the explosion action is of decisive importance. The crack formation exerts an influence on the change in physico-mechanical rock properties, particularly, the rock-mass permeability. As the permeability in the near-well zone of the productive strata rises, the producing well discharge increases. It is known that for intensifying the stratum permeability, acids, surface-active substances, and dissolvents, as well as different kinds of thermal treatment, etc. are widely applied [1, 2]. The method of controlled change in physico-mechanical properties of rocks in the near-well zone using the pulse action, including explosion energy is perspective [3].

At the present time, there are a number of models for investigating the explosion action exerted on solid medium [4 – 8]. The zone model is the most adequate, since it takes into account both the rock properties and the failure character [5]. For real conditions, the theoretical description of crack formation under explosion is a sufficiently complicated problem. This is associated with the fact that rock mass is inhomogeneous, and the blast loads are the highly intensive nonlinear actions causing the irreversible processes within the medium. Based on the solution of system of differential equations, the wave field can be determined only for certain model media [9 – 11]. For real media, it is impossible to calculate the parameters of propagating nonlinear disturbances and the change in the physico-mechanical properties in each case.

In this paper, the method is proposed for estimating the region of crack formation under the action of intensive wave loads. To characterize the rock-mass shattering in blast wave propagation, we selected the energy criterion. The investigations confirmed the functional dependence of failure zone size on the explosion energy. In addition to it, the obtained analytical relations indicate the blast wave characteristics which affect the failure process and lay the theoretical foundations for estimating the blast wave properties by the known failure zone.

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MODEL OF WAVE DISTURBANCE ATTENUATION

During wave disturbance propagation in the medium, the wave attenuates, which is governed not only by its geometrical divergence, but also by irreversible losses of energy spent for crack formation. Physically, the functional independence of the first and second processes is possible only in acoustical approximation. In general case, however, as the cracks form, the load is such that the waves are nonlinear. Therefore, we restrict ourselves to estimation of only the shattering zone. Yet for mathematical description of wave field, we assume that these two processes are functionally independent.

We suppose that for one-dimensional motion in medium without energy losses, the dependence of the blast wave front pressure p_f on the distance is determined by the relation:

$$p_f = A\left(\frac{Q^{1/\nu}}{r}\right)^{\mu}, \tag{1.1}$$

where Q is the explosion energy; the parameter ν determines the type of symmetry: $\nu = 1$, 2, 3 are the plane, cylindrical, and spherical symmetries, respectively; μ is the constant value; A is the dimension factor depending on both the medium and the explosive properties [9].

Assume that when the blast wave is at the distance r at the moment $t = t_1$, the pressure changes by the exponential law [9]:

$$p(r,t) = \theta(t - t_1)\, p_f(r) \exp\left(\frac{t_1 - t}{\tau}\right). \tag{1.2}$$

Here, $\theta(t)$ is Heaviside function. For a convenience, we accept $t_1 = 0$ in unambiguous cases.

The characteristic time of the wave load action without energy losses has the following functional dependence:

$$\tau(r,Q) = BQ^{1/\nu}\left(\frac{r}{Q^{1/\nu}}\right)^{\beta}, \tag{1.3}$$

where B is the dimension factor [9].

Since the displacement is insignificant as compared with the distance under investigation, we can state that relations (1.2) and (1.3) are valid for a particular medium microvolume, i.e., in the absence of energy losses, it undergoes pressure (1.2).

The values of the constants μ and β are interconnected. This fact can be established from the condition that the energy flow W through the closed surface is distance-independent:

$$r^{\nu-1} W(r) = \text{const}. \tag{1.4}$$

For weak blast waves, the flow energy through the surface unite:

$$W(r) = \int_{t_1}^{\infty} \frac{p^2(r,t)}{\rho c}\, dt, \tag{1.5}$$

where ρ is the medium density; c is the longitudinal wave velocity [12, 13]. With the value of $r^{\nu-1}W(r)$ required to be independent of distance, we obtain:

$$\beta = 1 - \nu + 2\mu. \tag{1.6}$$

Note that functional dependence (1.3) is known for water [9, 14]. At the same time, for geophysical medium, the empirical relationship (the spherical case) is used:

$$\tau = Q^{1/3} a + br \,. \tag{1.7}$$

The coefficients a and b are determined from the experimental data [3, 9, 11]. The calculation of τ by (1.3) and (1.7) showed that these values are close for $r \le 20 r_0$, where r_0 is the charge radius.

Relations (1.2) and (1.3) are obtained for blast wave without energy absorption by the medium, i.e., at condition (1.4). In general case, under intensive wave loads which are the blast waves, the irreversible nonequilibrium processes occur in the medium and lead to the additional wave attenuation. The experimental investigations indicate that the high-frequency disturbances attenuate faster than the low-frequency ones. We use one of the most frequently applied dependences characterizing the change in the spectrum density of pulse load [13, 15 – 17]:

$$S(r,\omega) = S_0(\omega) \exp(-\alpha |\omega| r) \,. \tag{1.8}$$

Physically, this relationship implies that the monochromatic wave of the frequency ω attenuates exponentially. In this case, the energy is absorbed by the medium and not redistributed between different wave frequencies. Usually the spectrum density of the wave disturbance is determined by Fourier transformation; for (1.2), it has the form:

$$S_0(\omega) = F\,[p(t)] = \int_{-\infty}^{\infty} \theta(t) p(t) \exp(i\omega t)\, dt = \frac{p_f}{\tau^{-1} - i\omega} \,. \tag{1.9}$$

REGION OF CRACK FORMATION

As a characteristic of rock shattering in blast wave propagation, the energy criterion is selected: the crack formation occurs until the energy absorbed by the medium exceeds a certain limiting value. In order to determine the energy remained in the medium after the blast wave propagation, we calculate first the total energy flow through the surface unit by formula (1.5):

$$W(r) = \frac{1}{\rho c} \int_0^{\infty} p^2(r,t)\, dt = \frac{p_f^2}{\pi \rho c} \int_0^{\infty} \frac{\exp(-2\alpha\omega r)}{\omega^2 + \tau^{-2}}\, d\omega \,. \tag{2.1}$$

The latter equality follows from Parseval theorem [18].

The value of the integral:

$$I(r,\omega) = \int_0^{\infty} \frac{\exp(-2\alpha\omega r)}{\omega^2 + \tau^{-2}}\, d\omega \tag{2.2}$$

can be expressed through the special functions. However, such notation is ponderous and uninformative. More effective is the approximation, where (2.2) is estimated by the saddle point method. From the appendix to this article, integral (2.2) has the following functional dependence on r and τ: $I = e^{-1}\sqrt{2\pi\tau}\,\mathrm{arctg}(\tau / 2\alpha r)$. Therefore, the whole energy flow to pass through the surface $\sigma(\nu) r^{\nu-1}$, where $\sigma(\nu) = 2\pi(\nu - 1) + (\nu - 2)(\nu - 3)$, equals:

$$G(r) \equiv \sigma(\nu) r^{\nu-1} W(r) = \sigma(\nu) D \frac{Q}{\rho c} \mathrm{arctg} \frac{\tau(r,Q)}{2\alpha r}, \quad D = A^2 B e \sqrt{\frac{2}{\pi}} \,. \tag{2.3}$$

Using (2.3), we can estimate the energy remained in the medium layer $\sigma(\nu)r^{\nu-1}dr$ after the blast wave propagation:

$$G(r) - G(r+dr) = -\sigma(\nu)\,\frac{d[r^{\nu-1}W(r)]}{dr}\,dr\,.$$

According to the accepted failure criterion, the crack formation takes place if the *k*-share of the energy exceeds a certain admissible value. Denote the maximum admissible energy of crack formation per volume unite by γ_ν and obtain the crack formation criterion:

$$-k\sigma(\nu)\,\frac{dr^{\nu-1}W(r)}{dr} \geq \sigma(\nu)r^{\nu-1}\gamma_\nu\,. \tag{2.4}$$

Note that in general case, the specific energy γ_ν of failure depends on *ν*, and the failure mechanism can vary. For example, the tangential stresses to be considerable in rock mass failure under the action of the cylindrical or spherical waves are absent in the plane case.

We proceed to the investigation of the coefficient *k*. As mentioned above, it has the following sense. The *k*-share of the blast wave energy absorbed by the medium is spent for crack formation. It is evident that in failure zone, the value of *k* can be distance-dependent. We can consider the energy γ_ν to be the same for different *Q* on the boundary of crack formation region. Therefore, it is assumed that values of k coincide in this case.

Substituting (2.3) into (2.4), we derive the equation connecting *Q* and the failure zone size r_b:

$$\frac{Q^{2-\frac{2}{\nu}\mu}\,r_b^{2(1+\mu-\nu)}}{(2\alpha r_b)^2+\tau^2(r_b,Q)} = \frac{\gamma_\nu}{H}\,, \quad H = e\sqrt{\frac{2}{\pi}}\,(1-\beta)\,\frac{\alpha A^2 B^2}{\rho c}\,. \tag{2.5}$$

By forward substitution, the solution of this equation takes the form:

$$r_b = RQ^{1/\nu}\,, \tag{2.6}$$

where the dimensional constant *R* satisfies the equation:

$$\frac{R^{2(1+\mu-\nu)}}{(2\alpha R)^2+(BR^\beta)^2} = \frac{\gamma_\nu}{H}\,. \tag{2.7}$$

Thus, the relations obtained make it possible to confirm the following statements. Firstly, equality (2.6) previously established experimentally is theoretically substantiated [13 – 20]. Also, the conformity of the theoretical and experimental results indicates the correct choice of the basic characteristics of the model proposed for the physical phenomenon of crack formation in a rock mass. Secondly, expression (2.7) enables us to estimate the coefficient *R* by the medium and explosive properties. Moreover, using (2.7), we can consider the inverse problem and calculate the blast wave parameters, as well as the medium properties by the known failure zone: to determine α, γ_ν, A, and *B* through the value of *R*.

ENERGY AND REGION OF CRACK FORMATION

Examine the change in region of crack formation at fixed explosion energy relative to γ_ν. Represent (2.7) in the dimensionless form:

$$\frac{l^{1-\nu-\beta}}{l^{2(1-\beta)}+1} = \Gamma\,. \tag{3.1}$$

Here, $\Gamma = \gamma_\nu B^{1-\nu/(1-\beta)} \rho c / (2e^{-1}\sqrt{2/\pi}(1-\beta) k A^2 \alpha^{\nu/(1-\beta)})$ and $l = (2\alpha/B)^{1/(1-\mu)} R$ are the dimensionless values. Figures 1, 2 present the graphs of the dependence $\Gamma(l)$ at different β for the cylindrical ($\nu = 2$) and spherical ($\nu = 3$) cases, respectively.

Rewriting (3.1) as $\Gamma^{-1} = l^\nu (l^{1-\beta} + l^{-(1-\beta)})$, we note that $\Gamma^{-1} = l^{\nu - |\beta - 1|}$ at $l << 1$, and $\Gamma^{-1} = l^{\nu + |\beta - 1|}$ when $l >> 1$. In logarithmic coordinates, they are the straight lines going through the point $l = 1$, at $\Gamma = 0.5$ (dark-colored points in Figs. 1, 2). Provided $\beta = 1$, both asymptotic forms coincide; if $\beta \neq 1$, the graphs are convex upwards. Pay attention to the fact that with $|\beta_1 - 1| = |\beta_2 - 1|$, the graphs coincide for any two values of μ_1 and μ_2 satisfying the relationship $\mu_1 + \mu_2 = \nu$. It is obvious from Fig. 1 that curve *3* corresponds both to $\mu = 0.5$ and $\mu = 1.5$. The same is observed in Fig. 2: curve *3* corresponds simultaneously to the values $\mu = 1$ and $\mu = 2$.

In the different domains $\Gamma > 1$ and $\Gamma < 1$, the same relative energy change required for failure of Γ leads to the various changes in size of the failure zone. For example, the decrease of Γ in the domain $\Gamma > 1$ by a factor of 2 causes more considerable increase in the failure zone as compared with the same decrease of Γ in $\Gamma < 1$.

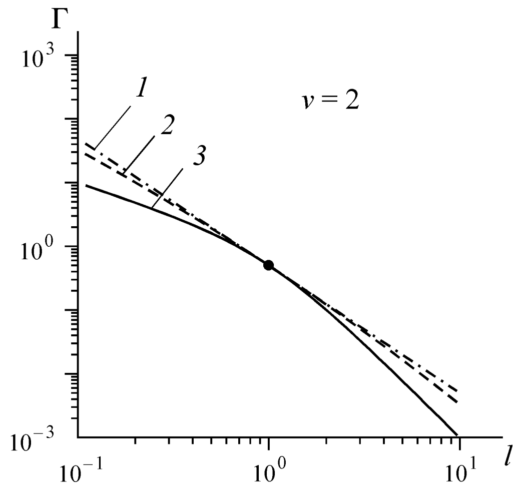

Fig. 1. Graphs of dependence $\Gamma(l)$: *1* — $\beta = 1$; *2* — $\beta = 1 \pm 0.44$; and *3* — $\beta = 1 \pm 1$

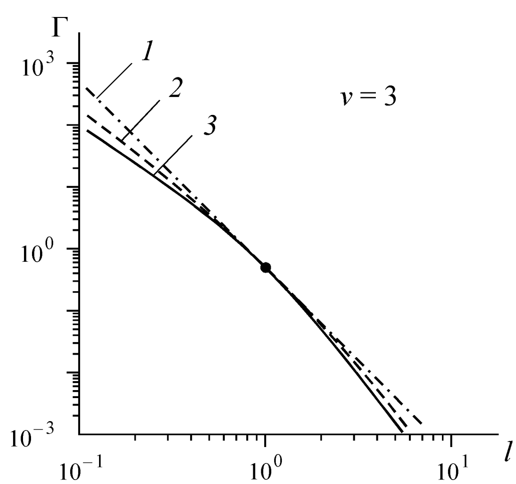

Fig. 2. Graphs of dependence $\Gamma(l)$: *1* — $\beta = 1$; *2* — $\beta = 1 \pm 0.74$; and *3* — $\beta = 1 \pm 1$

It is required that in real process, the conditions corresponding to the value close to $l = 1$ in terms of the dimensionless variables *l*, Γ are realized. For example, for blasting in granite, the values $2\alpha r_b$ and $\tau(r_b, Q)$ coincide completely, i.e., $l = 1$. This implies that the process is governed both by the characteristic time τ of the blast load and the wave attenuation at the distance $2\alpha r$. When $l << 1$, the process of crack formation is determined by τ, and conversely at $l >> 1$, the most important characteristic is the degree of the blast wave attenuation with distance.

COMPARISON WITH THE EXPERIMENT

Theoretical dependence (2.6) of the crack-formation region size on the blasting energy contains only one unknown parameter *R*. Thus, to find *R* and the dependence $r_b = r_b(Q)$, it is sufficient to carry out one experiment on blasting the solid medium; in the experiment, for assigned *Q*, the size r_b of the fractured region is known. The comparison of the experimental results [13, 19] and (2.6) is shown in Fig. 3. To calculate *R*, we specified a fiducial point whose values for granite and limestone are presented in Table 1. In logarithmic coordinates, the graph of (2.6) represents a straight line passing through a fixed point. The fiducial points are darkened in Fig. 3.

The blast wave parameters are presented for trotyl. The dimension factor *B* is established from the condition of coincidence of the values of τ calculated by (1.3) and (1.7) at a distance equal to the charge radius. In (1.1), the dimensional constant *A* is connected with A' from Table 1 by the relation: $A' = A(Q_0^{1/3}/r_0)^{\mu}$, where $Q_0 = 1$ kg, $r_0 = 0.054$ m.

In fact, equality (2.6) is not a new result. However, the identity of the theoretical dependence $r_b \sim Q^{1/3}$ and the experimental data indicates that the assumptions in the model are admissible. In this case, *R* functionally depends on ρ, *c*, α, μ, *A*, *B*, and γ_v (but without *Q*), i.e., on the properties of both the medium and the explosive. Using (2.7), we determine *R* by the above-mentioned values as well as obtain γ_v or another value from the listed ones by the known *R*, which corresponds to the direct and inverse problems, respectively.

For inverse problem, the energy density γ_v / k is estimated. It is absorbed by the medium near the boundary of crack formation region. The γ_v / k ratio obtained from (2.7) is presented in Table 1.

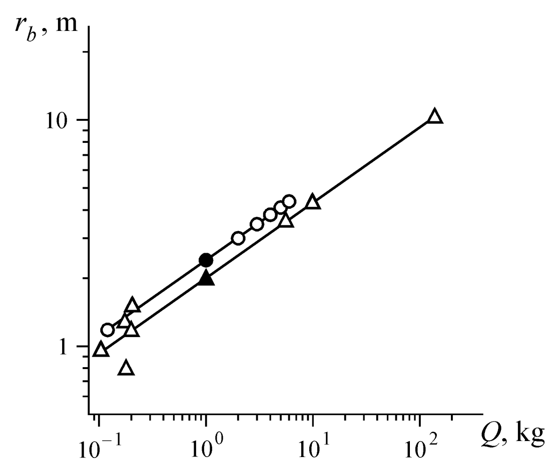

Fig. 3. Size of crack formation region as explosion energy function (theoretical results — straight lines; experiment: ○ — granite, △ — limestone)

TABLE 1

Medium	Features			Blast wave				Fixed point		Calculation	
	ρ, kg/m^3	c, m/s	$\alpha \cdot 10^5$, m/s	μ	$A' \cdot 10^{-8}$, Pa	$a \cdot 10^5$, s/kg$^{1/3}$	$b \cdot 10^4$, s/m	Q, kg	r_b, m	R, m/kg$^{1/3}$	γ_v / k, J/m^3
Granite	2600	5720	2	1.13	1.09	3.6	0.18	1	2.4	2.4	0.67
Limestone	2580	4650	6	1.13	0.30	4.1	4.69	1	2.0	2.0	0.19

At the same time, the direct problem of calculating R by the energy γ_v spent for crack formation makes it possible to use additional theoretical and experimental results which are not pertinent to explosion. In this case, apart from the well-known values ρ, c, μ, A, B, and α [15–17] (Table 1), it is required to find γ_v and k or γ_v / k. Knowing γ_v and k, we can estimate R by (2.6) without carrying out an experiment. Currently, the problem on finding the values of γ_v and k from the other theoretical and experimental data remains unsolved.

CONCLUSION

The model of crack formation under the action of blast wave is developed. It takes into consideration the change in the wave load to be governed by geometrical divergence of wave as well as irreversible losses in the medium. The energy criterion was selected for the solid medium failure during blast wave propagation. The geometrical similarity of failure zone caused by the explosion energy is proved. The conformity of the theoretical and experimental results indicates the correct choice of the basic characteristics for the proposed model of crack formation in a rock mass. On the basis of the assigned features of the medium and explosive (the direct problem), the analytical dependences ensure the estimation of crack formation zone, as well as lay the foundation for determining the blast wave parameters and the medium properties by the known failure zone (the inverse problem).

APPENDIX

Reduce integral (2.2) to the form convenient for applying the saddle point method:

$$I = \int_0^\infty d\omega \, \frac{\exp(-2\alpha\omega r)}{\omega^2 + \tau^{-2}} = \tau \operatorname{arctg} \tau\omega \exp(-2\alpha\omega r) \Big|_0^\infty + 2\alpha r \tau \int_0^\infty d\omega \operatorname{arctg}(\tau\omega) \exp(-2\alpha\omega r) =$$

$$= \tau \int_0^\infty d\omega \, \frac{\operatorname{arctg}(\tau\omega)}{\omega} \exp(-2\alpha\omega r + \ln(2\alpha\omega r)).$$

At the point $\omega_1 = (2\alpha r)^{-1}$, the exponential curve has the maximum. In the vicinity of this point, the integrand contributes to the integral required, and according to the saddle point method, it has the form:

$$I \approx 2\alpha r \tau \operatorname{arctg}(\tau\omega) \int_{-\infty}^\infty d\omega \exp(-1 - 2\alpha^2 r^2 (\omega - \omega_1)^2) = \frac{\sqrt{2\pi}}{e} \tau \operatorname{arctg} \frac{\tau}{2\alpha r}. \qquad \text{(a. 1)}$$

The numerical calculations show that the deviation of the exact value of integral (2.2) from approximate value (a. 1) does not exceed 8 %.

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