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EVOLUTION OF STRONG SHOCK WAVES IN A MEDIUM WITH THERMAL RELAXATION

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In the classical case where there are no relaxation processes at the front of a shock wave, the nonsteady motion of a medium induced by the instantaneous release of energy at a point is described by a self-modeling solution [7, 10]. However, when there are relaxation processes whose characteristic time is comparable to the propagation time of the wave, the parameters on the front depend significantly on how completely the medium has relaxed. Of particular interest are those relaxation processes in which the pressure in the medium drops.

The present paper is concerned with the effect of thermal relaxation at the wave front on the evolution of a shock wave. An example of a medium where relaxation properties are significant is a two-phase medium. For example, the attenuation of shock waves in gas-liquid foams generated by concentrated high voltage can be described in terms of a relaxed discharge of heat from the gas phase into the condensed phase [1, 2].

The nonsteady one-dimensional motion of a continuous medium can be described in terms of the hydrodynamical equations in the Euler formulation (all notation is as generally accepted)

$$\frac{\partial\rho}{\partial t}+\frac{1}{r^2}\cdot\frac{\partial r^2\rho u}{\partial r}=0;\qquad \frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+\frac{1}{\rho}\frac{\partial p}{\partial r}=0; \tag{1}$$

$$\frac{\partial}{\partial t}\left(\rho E+\rho\frac{u^2}{2}\right)+\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2u\left(\rho E+\rho\frac{u^2}{2}+p\right)\right]=0.$$

This system of equations is supplemented by the balance equation of the total energy; the energy of the medium bounded by the shock wave is equal to the initial energy of the medium $E(p_0, \rho_0)$ and the energy of the explosion E_0. For an intense wave $E_0 \gg E(p_0, \rho_0)$ and then we have

$$4\pi\int\limits_0^{r_f}\left(\rho E+\rho\frac{u^2}{2}\right)r^2dr=E_0. \tag{2}$$

The parameters with subscript 0 refer to the unperturbed medium.

The system of equations is closed with the help of the dynamical equation of state

$$\tau_0\frac{d}{dt}\left[E-\frac{p(1-\varepsilon)}{\rho(\gamma-1)}\right]+\left[E-\frac{p(1-\varepsilon)}{\rho(\Gamma_0-1)}\right]=0, \tag{3}$$

where τ_0 is the characteristic thermal relaxation time, ε is the volume fraction of the condensed phase, γ is the adiabatic exponent of the gas, and Γ_0 is the equilibrium adiabatic exponent of the medium.

The dynamical equation of state is based on nonequilibrium thermodynamics [2, 11]. Assuming that a loss of thermal equilibrium occurs as a result of the shock compression of the medium and afterward there is a gradual return to thermodynamic equilibrium with a char-

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acteristic time constant τ_0, we obtain from (3) an algebraic equation of state for the shock wave flow in the form

$$E=\frac{p(1-\varepsilon)}{\rho(\Gamma-1)};\qquad \Gamma=\Gamma_0+(\gamma-\Gamma_0)\exp\left(-\frac{\tau'}{\tau_0}\right). \tag{4}$$

Here τ' is the time referenced to a microvolume in the wave. It satisfies the differential equation

$$\frac{\partial\tau'}{\partial t}+u\frac{\partial\tau'}{\partial r}=1. \tag{5}$$

The parameter Γ is physically a relaxation parameter and determines how the thermodynamic quantities of the medium are related during the relaxation process. It can vary between $\Gamma = \gamma$, characterizing flow where the thermal processes are frozen, and $\Gamma = \Gamma_0$, corresponding to equilibrium flow. For bulk processes we have $\Gamma_0 < \gamma$. We note that for two-phase gas-containing mixtures, Γ_0 is usually close to 1, though not exactly equal to it. Below we assume $\varepsilon = 0$; when $\varepsilon \neq 0$ one can use the method of [3].

The boundary conditions for the case of an intense explosion are found from the Renkin–Giugonio conditions with the assumption that the thermal relaxation is frozen on the shock front, i.e., for $r = r_f$

$$p=\frac{2}{\gamma+1}\rho_0D^2;\qquad \rho=\frac{\gamma+1}{\gamma-1}\rho_0;\qquad u=\frac{2}{\gamma+1}D;\quad \tau'=0,$$

and $u = 0$ when $r = 0$. The initial conditions are given by the expression $\tau' = 0$ and by the self-modeling solution of the system [7, 10], which is correct in the absence of relaxation processes. Here D is the velocity of the shock wave.

The system of Eqs. (1) through (5) is first made dimensionless as follows:

$$V=u/D;\qquad P=p/p_0;\qquad R=\rho/\rho_0;\qquad \tau=\tau'/\tau_0;$$
$$\eta=r/r_f;\qquad \chi=r_f/\tau_0D, \tag{6}$$

and $\tau_0\dfrac{\partial\chi}{\partial t}=1-z;\ z=\dfrac{r_f}{D^2}\dfrac{dD}{dt}$. In the limiting case of no relaxation processes (Γ = const) the dimensionless variables R, V, P are self-modeling. In addition, $z(0) = -3/2$, which corresponds to the initial instant of time.

$$\psi(\chi)=\int_0^1\left(\frac{P}{\Gamma-1}+\frac{1}{2}RV^2\right)\eta^2d\eta,$$

we obtain the equation

$$\chi(1-z)\frac{\partial\psi}{\partial\chi}+(2z+3)\psi=0. \tag{7}$$

The system of Eqs. (1) and (5), in which the variable E has been eliminated with the help of (3), has the following form in terms of the new variables

$$\chi(1-z)\frac{\partial R}{\partial\chi}+(V-\eta)\frac{\partial R}{\partial\eta}+R\left(\frac{\partial V}{\partial\eta}+\frac{2V}{\eta}\right)=0; \tag{8}$$

$$\chi(1-z)\frac{\partial V}{\partial\chi}+(V-\eta)\frac{\partial V}{\partial\eta}+zV+\frac{1}{R}\cdot\frac{\partial P}{\partial\eta}=0;$$

$$\chi(1-z)\frac{\partial P}{\partial\chi}+(V-\eta)\frac{\partial P}{\partial\eta}+2zP+\Gamma P\left(\frac{\partial V}{\partial\eta}+\frac{2V}{\eta}\right)+\frac{\Gamma-\Gamma_0}{\Gamma-1}\chi=0;$$

$$\chi(1-z)\frac{\partial\tau}{\partial\chi}+(V-\eta)\frac{\partial\tau}{\partial\eta}=\chi.$$

The boundary conditions for this system take the form

$$R=\frac{\gamma+1}{\gamma-1};\quad P=V=\frac{2}{\gamma+1};\quad \tau=0\quad (\eta=1);$$

$$V=0\quad (\eta=0).$$

We note that the system of quasilinear Eqs. (8) is hyperbolic in the region $\{\chi > 0, 0 < \eta < 1\}$. Indeed, the characteristic roots of the determinant formed by the coefficients of the partial derivatives with respect to η are real and, even though they are multiple, there exists a transformation with a nonzero determinant which reduces the system of differential equations to canonical form.

Shock wave flow in a medium with thermal relaxation was studied in [1, 2] using the system of equations derived above. An approximate analytical solution was obtained in [1] for the parameters on the shock front, and in [2] a solution of the system of equations at small times $t < \tau_0$ was found with the help of numerical methods.

For times less than a certain value $t^* < \tau_0$ the system of equations can be solved by the method of [2], while for $\tau > t^*$ the differential Eqs. (7) and (8) can be approximated to first order by an implicit finite-difference scheme of the triangular type (running calculation). The calculated is started on the shock front, where all the flow parameters are known. The implementation of the implicit scheme is complicated by the nonlinearity of the equations and therefore the variables were found using the method of [5], adopted for the case of two independent variables. As the calculation proceeds, it is essential that the integral relation (7) and the boundary condition $V = 0$ be satisfied simultaneously. By varying the value of z, these two requirements can be satisfied using the method of successive approximations. A singularity of the saddle-point type near the center of symmetry was crossed by adjusting z and then by the following method. Beginning with a certain η^*, when the lowest values of the density are reached, the value of the velocity is extrapolated according to the linear form $V = V(\eta^*)\eta/\eta^*$, in order that significant errors not be carried into the kinetic energy.

The stability and convergence of the difference scheme is rather difficult to study analytically. In order to study these properties of the difference scheme, we considered a finite-difference scheme based on successive determination of different values of the flow parameters. The equation was written such that on one time step the scheme was stable with respect to the coordinate. Comparison of the results for the two schemes and the good agreement of the results indicates that the difference schemes are applicable. Calculations with variable directions are more complicated and require more execution time; this was taken into account in the final choice of the finite-difference scheme.

In order to suppress possible high-frequency computational oscillations in the finite-difference procedure, we used the smoothing method of [6]. The resulting smoothing coefficients (0.002 for the density and 0.005 for the pressure and velocity) lead to the desired result and do not carry errors into the flow parameters. A FORTRAN program was written for this procedure for the ES computer.

Graphs showing the basic features of the evolution of strong shock waves in a relaxing medium are shown in Figs. 1 through 4. The effect of the relaxation process on the change of the pressure profile is shown in Fig. 1 ($\gamma = 1.4$; $\Gamma_0 = 1.2$; 1 – $t/\tau_0 = 0$; 2 – 1; 3 – $t/\tau_0 = 6$). In the central region a decrease in P is observed at the initial instant of time, but later, as the average thermodynamic equilibrium is reached, the pressure increases and approaches the value corresponding to self-modeling flow with Γ_0.

The parameter Γ giving a relation of the form (4), can be considered as an interfacial relaxation function for the case of a two-phase medium. When thermodynamic equilibrium exists between the phases Γ_0 is identical to the ratio of the specific heats of the medium at constant pressure and volume, determined assuming additivity [9]. In this case a decrease in Γ_0 is equivalent to an increase in either the concentration of the condensed phase, or its specific heat, given that the other conditions are the same. In performing the calculations it was assumed that over the entire range of Γ_0, the quantity τ_0 remains constant.

The presence of the relaxation process leads to a qualitative change in the nature of the attenuation of the shock wave. Unlike the case of a nonrelaxing medium, where the pressure and propagation velocity of the shock wave vary according to the power laws ($p \sim r_f^{-s}$, $D \sim r_f^{-0.5s}$) with a constant exponent $s = 3$, the presence of relaxation leads to a

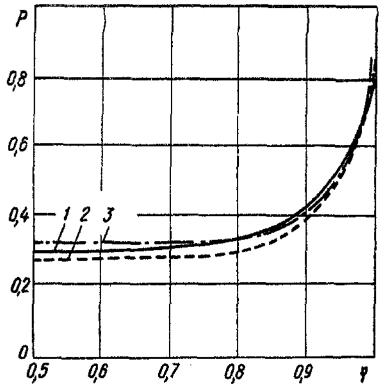

Fig. 1

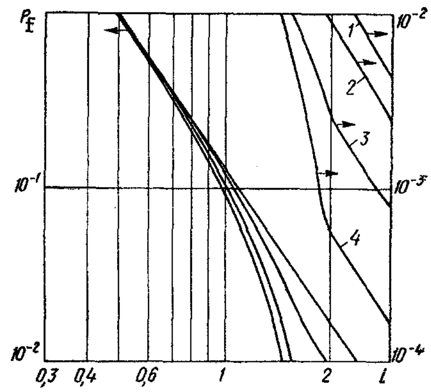

Fig. 2

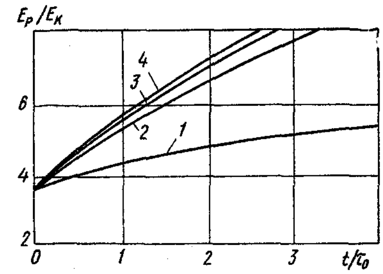

Fig. 3

variation of the exponent. The maximum absolute value s_{max} is reached (for the other parameters held constant) later in a medium with a smaller value of Γ_0. In Table I we show the dimensionless time t/τ_0 at which the maximum value s_{max} is reached for specific values of Γ_0 and for fixed $\gamma = 1.4$. Far from the energy source and for $t \gg \tau_0$, the relaxation processes do not really affect the nature of damping of the velocity and pressure on the shock front. In this case s asymptotically approaches its limiting value, corresponding to the self-modelling solution. This is illustrated in Fig. 2 ($\gamma = 1.4$; 1 – $\Gamma_0 = 1.4$; 2 – 1.2; 3 – 1.05; 4) $\Gamma_0 = 1.01$) where the dependence of the pressure on the wave front $p_f = p\tau_0^{6/5}(\rho_0 A^2)^{-1}$ is shown as a function of the dimensionless distance $\ell = r_f(\tau^{2/5}A)^{-1}$. As seen from the graphs, under the assumptions made here, the curves approach the asymptotic value at the same distance from the energy source. We note that the increase of the exponent s above the self-modeling value is purely a relaxation effect. This effect is characteristic of the propagation of strong shock waves for a wide class of multiphase media (foam [1, 2], soil [8], bubble-like materials [4]).

An important characteristic related to the amount of attenuation of the shock wave in the medium is the variation of the ratio of the internal energy of the medium to its kinetic energy during the relaxation process. The dependence of the energy redistribution from the explosion between the internal and kinetic energies of the medium on the dimension-

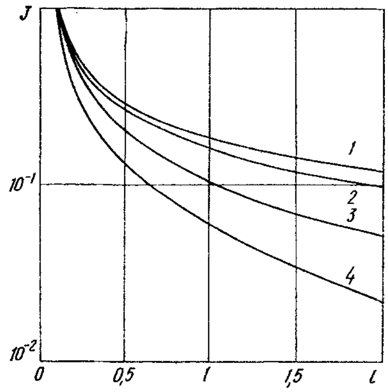

Fig. 4

TABLE 1

Γ_0	1,2	1,05	1,01
s_{max}	3,6	6,6	12,2
t/τ_0	3,2	5,4	7,8

less time is shown in Fig. 3 (γ = 1.4; 1 – Γ_0 = 1.2; 2 – 1.05; 3 – 1.01; 4 – Γ_0 = 1.0). We see that the energy redistribution in time also reaches a limiting value corresponding to thermodynamic equilibrium in the medium, and this process occurs more rapidly the larger the value of the parameter Γ_0. However at any fixed instant of time, in a medium with a smaller value of Γ_0 the ratio of the internal energy to the kinetic energy is always larger. But with decreasing Γ_0 the differences in the energy redistribution decrease and finally the ratio of energies approaches the dependence corresponding to the limiting value Γ_0 = 1 (curve 4).

The attenuation of shock waves in a relaxing medium is intimately connected with the need to decrease in their intensity in order to avoid the breakup of the structure. In practice, breakup is determined in most cases either by the impulse of the shock wave, or by a quantity involving the impulse and the pressure on the front [12]. Since relaxing multiphase media are so widely used in the damping of shock waves, knowledge of how the impulse of the shock changes during relaxation processes is of interest. This is shown in Fig. 4 (γ = 1.4; 1 – Γ_0 = 1.4; 2 – 1.2; 3 – 1.05; 4 – Γ_0 = 1.01) as the dependence of the dimensionless impulse of the pressure

$$J = I\tau_0^{1/5}(\rho_0 A)^{-1}; \qquad I = \int_{r_1}^{\infty} p(r_1, t)\,dt$$

on the distance ℓ. For constant ρ_0, a decrease of the parameter Γ_0 leads to a decrease of the impulse of the pressure at a fixed relative distance and the amount of the decrease grows with increasing specific heat of the medium. It must be pointed out that, all other things being equal, an increase in the density of the medium (as shown in Fig. 4) can lead to an increase in the impulse of the pressure above the value for a less dense nonrelaxing medium, in spite of the decrease in the peak pressure (Fig. 2).

Therefore the results obtained above show the features of the attenuation of shock waves in the presence of relaxation processes in the medium. Our approach can be used to describe attenuation of shock waves in different kinds of gas-containing materials. The effect of the volume fraction of the condensed phase can be taken into account with the help of the method of [3].

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PROBABILISTIC MODIFICATION OF THE THEORY OF FRIABLE MEDIA

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Vertical prismatic and cylindrical shells (silos, chemical equipment, and other vessels) are widely used in engineering for the storage and treatment of friable materials. The pressure of the friable filling is the basic load on the walls of such shells. Therefore, evaluating such pressures is an important problem of the mechanics of friable media. Numerous theoretical works have considered its solution; see the reviews in [7, 17]. A common feature of these investigations is that, beginning with the pioneering work [16] and ending with the latest theories [1, 15], they are all undertaken within the framework of a deterministic approach, which allows the average pressure of the friable filling to be estimated with a particular degree of accuracy, but does not take account of the random scattering of these values, which is observed in the predominant majority of experiments; see [8, 12, 23] and others.

In the present work, a probabilistic approach to calculating the static pressure of friable filler at the shell walls is considered. The approach is based on a modification of Janssen theory [16], in which one of the determining parameters (the lateral thrust coefficient) is not constant but forms a random field. On the basis of the correlational theory of random functions, the probabilistic characteristics of the filler pressure are investigated. The results are compared with experimental data.

1. Formulation of the Problem

Consider a prismatic (cylindrical) shell with absolutely rigid vertical walls and a horizontal floor; the shell is filled with friable material. Let Σ denote the tensor field of intrinsic static loads arising in the filler after filling the vessel. Let σ_z and σ_n be the vertical and horizontal normal components of the field Σ at the lateral surface S of the filler and $\tau \in \Sigma$ be the vertical tangential load at S. The problem is to determine these loads.

Note that accurate solution of this problem is associated with a series of significant difficulties, of which two may be indicated here. First, the determining equation for the

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