$$\frac{K_0^2}{2} \nu \tau_r < Z_r (2j + 2\Delta j + 1)\, e^{-\frac{B}{T}(j+\Delta j)(j+\Delta j+1)}, \tag{16}$$

be satisfied, where Z_r is the rotational partition function. Condition (16) is realized for a large fraction of absorption bands, with the exception of the far wings corresponding to large j.

The quantity W_0 characterizes the maximum rate of energy input into the system. Since all the energy introduced changes into translational degrees of freedom, it is possible to consider W_0 as the limiting rate of vibrational relaxation.

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ENERGY CHARACTERISTICS OF A CARBON MONOXIDE GASDYNAMIC LASER

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The characteristics of gasdynamic lasers based on mixtures of carbon monoxide with nitrogen and inert gases were investigated and the populations of vibrational levels of CO molecules, the gain of the mixture, and the generation power were determined in [1-8]. But the parameters of a gasdynamic laser (GDL) in the optimum emission mode have not been determined up to now. The difficulties in calculating the optimum energy characteristics are connected with the complexity of the calculating model and the large number of parameters of the system. The energy characteristics of a CO gasdynamic laser are calculated and optimized in the present report on the basis of a simple model.

1. Calculating Model. Let us consider the escape of a binary gas mixture $\psi_C CO + \psi_N N_2$ (ψ_C and ψ_N are the molar fractions of CO and N_2, respectively) from the flat supersonic nozzle of a gasdynamic laser having a critical cross section with a height h_* and an initial aperture half-angle φ. At a degree of expansion S_0/S_* the expanding part of the nozzle changes into a plane-parallel section where the optical resonator is mounted.

We make the following assumptions, permitting a simplified calculation of the energy characteristics of a CO gasdynamic laser:

1. Losses of vibrational energy as a result of V–T processes occur mainly in the initial section of supersonic escape near the critical cross section of the nozzle.

2. The time the gas spends in the resonator exceeds the characteristic time of establishment of a quasi-steady distribution of molecules over the vibrational energy levels.

3. We consider a plane-parallel Fabry–Perot resonator which forms a nondiverging light flux in a geometrical treatment.

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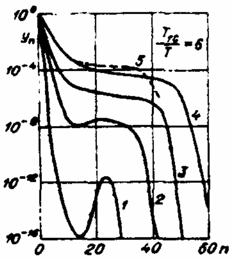

Fig. 1

The justification of assumptions 1 and 2 emerges from the following model of the physical processes occurring in a CO GDL.

With gas flow in a Laval nozzle the freezing in of the vibrational energy of the gas occurs in the expanding section of the nozzle in some cross section S_1 not known in advance. Above the freezing-in cross section the gas is in a state close to thermodynamic equilibrium, while below the cross section S_1 the vibrational temperatures of the nitrogen and carbon monoxide molecules exceed the translational temperature of the gas. As the gas moves down from cross section S_1 in the expanding section of the nozzle a quasisteady distribution (corresponding to the frozen-in energy E_f and the translational temperature T of the gas in the given nozzle cross section) of the molecules over the vibrational levels, differing in general from a Boltzmann distribution, is formed in the gas under the conditions of the vibrational nonequilibrium of the anharmonic molecules. With a decrease in the pressure and translational temperature of the gas, the rate of vibrational-vibrational exchange falls sharply, and at some cross section $S_2 > S_1$ the characteristic time τ_0 of expansion of the gas stream becomes comparable with the characteric time τ_V of establishment of a quasisteady distribution of the molecules over the vibrational levels τ_V [9, 10]:

$$\left.\frac{\tau_0}{\tau_V}\right|_{S=S_2} = \text{const} \sim 1.$$

In this cross section the freezing-in of the distribution of the molecules over the vibrational levels occurs, i.e., at $S > S_2$ the distribution function F of the molecules over the vibrational levels is determined by the reserve of vibrational energy E_f of the gas and by the translational temperature $T(S_2)$ at the cross section S_2,

$$F = F[E_f(S_2),\ T(S_2)] \equiv F[T_{1N}(S_2),\ T_{1C}(S_2),\ T(S_2)],$$

where T_{1N} and T_{1C} are the vibrational temperatures of the first levels of nitrogen and carbon monoxide molecules, which are not equal to each other in general.

In the process of expansion the translational temperature of the gas declines, while the ratio of the vibrational to the translational temperature of the molecules increases. In the process the losses of vibrational energy can grow, generally speaking, owing to the rapid deactivation of molecules at high vibrational levels.

Estimating calculations which were made showed that when short nozzles ($\varphi \sim 30°$, $h_* \sim 1$ mm) are used the working gas is in slight nonequilibrium ($T_{1N}/T \leq 6$, $T_{1C}/T \leq 6$) in the region of $S_1 < S$ for a pressure $p_* <$ 1000 atm and a temperature $T_* < 2000°$K. In this case the losses of vibrational energy as a result of V−T and V−V processes, dependent on effects of the anharmonism of the molecules, do not exceed 10%.

In order that the losses of vibrational energy in the plane-parallel section of the nozzle in the resonator region also be small in comparison with the energy losses above the cross section S_1 it is necessary that the condition of slight nonequilibrium be satisfied in this region. This condition is assured by an efficient optical resonator which limits the high population of the upper vibrational levels of the molecules.

The amount of energy E of stimulated emission extracted from the resonator per unit mass of gas is

$$E = \eta(E_f - E_r), \tag{1.1}$$

where E_f is the frozen-in vibrational energy at the entrance to the resonator; E_r is the residual vibrational energy of the molecules at the exit from the resonator; $\eta \approx t/(2a + t)$ is the coupling coefficient, determining the ratio between the extracted energy and the energy absorbed by the mirrors in the resonator; a and t are the coefficients of absorption and transmission of the resonator mirrors, respectively (a, $t \ll 1$).

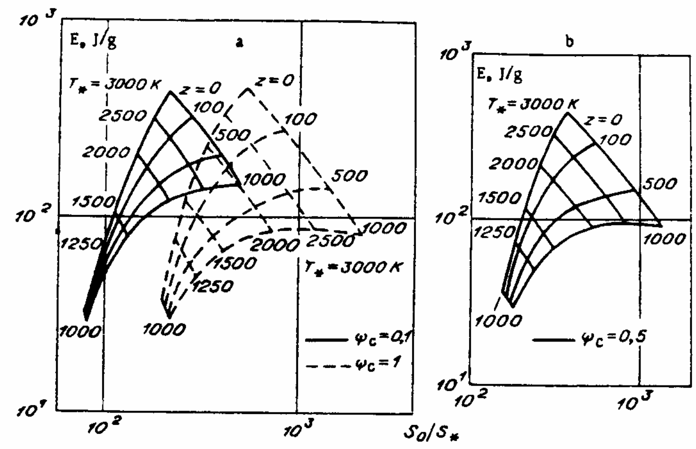

Fig. 2

The frozen-in vibrational energy was calculated by numerical integration of the system of differential equations of vibrational kinetics [11], with experimental values of the rate constants of the relaxation of vibrational energy of CO and N_2 molecules [2, 12-14] being used.

From an analysis of the relaxation equations it follows that for a gas mixture of a given composition, having a temperature T_* and a pressure p_* at the critical cross section of the nozzle, the frozen-in vibrational energy per unit mass of gas is a function of the two parameters T_* and $z = p_* h_* / \tan \varphi$.

The residual energy E_r of a mixture of a given composition $\psi_C CO + \psi_N N_2$ is a function of the vibrational temperatures T_{1C} and T_{1N} and the translational temperature at the exit from the resonator: $E_r = E_r(T_{1C}, T_{1N}, T)$. The values of T_{1C} and T_{1N} are found from the condition that the conversion of vibrational energy of the CO molecules into energy of stimulated emission in the resonator under consideration takes place so long as the maximum gain of the medium on vibrational–rotational transitions at the exit from the resonator under the conditions of a low-intensity light field does not fall to some limiting value α_0 determined by the resonator parameters. At the exit from the resonator one can write

$$\alpha(T_{1C}, T, p_1) = \alpha_0 = (2\bar{a} + \bar{t})/2, \qquad (1.2)$$
$$\Theta_N/T_{1N} - \Theta_C/T_{1C} = (\Theta_N - \Theta_C)/T,$$

where Θ_N and Θ_C are the characteristic temperatures of the N_2 and CO molecules; p_1 is the gas pressure in the resonator region; $\bar{a} = a/L$; $\bar{t} = t/L$; L is the length of the active zone of the resonator across the stream.

Determining T_{1C} and T_{1N} from (1.2), one can obtain an expression for the residual vibrational energy of the gas in the form of a function of four parameters

$$E_r = E_r(T, \bar{t}, \bar{a}, p_1). \qquad (1.3)$$

The gain of the P branch of the vibrational–rotational transition $n \to n-1$, $J-1 \to J$ is

$$\alpha = A \nu R G \left(\frac{y_{n,J-1}}{2J-1} - \frac{y_{n-1,J}}{2J+1} \right),$$

where A = const; ν is the transition frequency; $y_{n,J-1}$ and $y_{n-1,J}$ are the population densities of the upper and lower levels of the transition; R is a matrix element of the transition; G is a form factor; n and J are the vibrational and rotational quantum numbers corresponding to the transition at which the gain of the medium is maximal.

The populations of the vibrational levels of carbon dioxide and nitrogen molecules were calculated from the solution of the system of kinetic equations [2, 3, 15, 16] for 60 vibrational levels of CO and N_2 molecules on the basis of the assumption that a quasisteady distribution of the molecules exists at the exit from the resonator.

Mixtures of CO + N_2 with $\psi_C = 1$, 0.5, and 0.1 in the range of variation of the translational temperature from 100 to 500°K were considered.

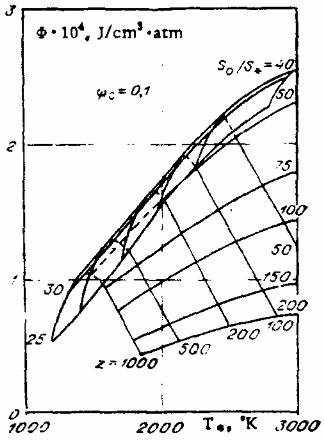

Fig. 3

The values of the probabilities of V–V and V–T processes used in the calculations are taken from [2, 12-14].

The results of calculations of the populations y_n of the vibrational levels of CO molecules ($\psi_C = 1$) at $T_{1C}/T = 6$ are presented in Fig. 1. Curves 1-4 correspond to translational gas temperatures T = 100, 200, 300, and 450°K, respectively. A calculation made in [16] for a temperature of 450°K is given for comparison (curve 5).

Using an expression in the form of (1.3) for the residual vibrational energy and representing the translational temperature and pressure of the gas in the form $T = T_* f_1(S_0/S_*)$ and $p_1 = p_* f_2(S_0/S_*)$ under the assumption of a constant adiabatic index $\gamma = 1.4$ for the frozen-in stream, we find that for a mixture of the given composition the energy of stimulated emission extracted from the resonator (Eq. (1.1)) is determined by the value of six parameters:

$$E(T_*, z, S_0/S_*, p_1, \bar{t}, \bar{a}) = \eta(\bar{t}, \bar{a})\,[E_f(T_*, z) - E_r(T_*, S_0/S_*, p_1\bar{t}, \bar{a})]. \qquad (1.4)$$

In calculating E we ignored the kinetics of the V–V and V–T relaxation processes and the interaction of molecules with the electromagnetic field of emission, and therefore the length of the resonator along the stream does not enter into (1.4) as a parameter of the problem.

In calculating the emission energy of a GDL we consider two variants:

1. The length of the GDL resonator is not limited by the conditions of the problem; the pressure in the resonator region is greater than 40 mm Hg. The characteristic distance l in which a quasisteady distribution of the molecules over the vibrational levels is formed, which determines the length of the resonator, will equal 30 cm in pure carbon monoxide at $p_1 = 100$ mm Hg and T = 300°K and will equal 300 cm at $p_1 = 100$ mm Hg and T = 160°K. At $p_1 = 200$ mm Hg the value of l is 1 and 1.5 m, respectively.

2. The length of the resonator is limited. No conditions are imposed on the pressure in the resonator.

2. A GDL with a High Gas Pressure in the Resonator Region.

When $p_1 > 40$ mm Hg the number of parameters being optimized can be reduced to one, since at these pressures the gain of the medium is determined mainly by the collisional mechanism of broadening of spectral lines and hence does not depend on p_1, and therefore p_1 is not an independent parameter.

We take T_* and z as the free parameters. We set the parameter $\bar{a} = a/L$ as equal to 10^{-4} 1/cm, which corresponds, for example, to a resonator with mirrors having a coefficient of absorption of 1% and to an active zone with a size of 1 m across the stream.

We will seek the maximum of the function $E(T_*, S_0/S_*, p_1, \bar{t}, \bar{a})$ with respect to the parameters $\bar{t}$ and S_0/S_*.

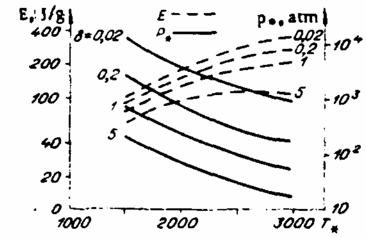

Fig. 4

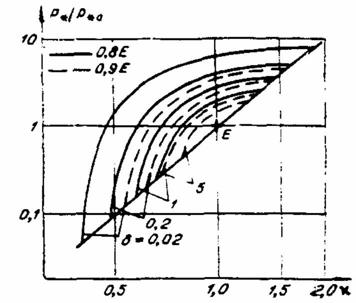

Fig. 5

The calculations show that, within the framework of the model under consideration with allowance only for the collisional mechanism of broadening of spectral lines, with an increase in S_0/S_* the specific energy of stimulated emission per unit mass of gas, with fixed z and T_*, emerges into a saturation section, reaching a maximum E_{**} as $S_0/S_* \to \infty$.

We limit S_0/S_* to values at which $E(z, T_*, \bar{a}, S_0/S_*) = 0.9\ E_{**}$. We designate this value of S_0/S_* as $(S_0/S_*)_0$ and call it the optimum value. In the case of $S_0/S_* < (S_0/S_*)_0$ and $p_1 > 40$ mm Hg the energy will decline sharply, while when $S_0/S_* > (S_0/S_*)_0$ the variation in E, found with allowance for both the collisional and the Doppler mechanism of line broadening, does not exceed 10%.

The results of a calculation of the specific energy E, optimized with respect to the parameters $\bar{t}$ and S_0/S_*, are presented in Fig. 2a, b in the form of a family of intersecting curves of $E = f[(S_0/S_*)_0]$ with T_* = const and z = const for binary mixtures of $CO + N_2$ with $\psi_C = 0.1$, 0.5, and 1. The numbers by the lines are the values of T_* and of z, atm · cm. Each value of E corresponds to an optimum value of $\bar{t}$ lying in the interval of $10^{-3}\ cm^{-1} < \bar{t} < 10^{-2}\ cm^{-1}$, with the energy having a weakly expressed maximum (within limits of 10% variation of E) in this interval of $\bar{t}$.

Using the graphs of Fig. 2, for any set of parameters T_*, z we can find the optimum degree of expansion $(S_0/S_*)_0$ and the corresponding value of the emission energy.

In comparing the results of the calculation for different CO concentrations, one can see that with a decrease in ψ_C the region of equal energies shifts toward smaller $(S_0/S_*)_0$, and it proves possible to make a CO gasdynamic laser with not overly high degrees of expansion (~ 100) and without the addition of inert gases, which increase the γ of the mixture.

In the region of $0.1 \le \psi_C \le 1$ with T_* = const and z = const ($z \neq 0$) the specific energy E grows with a decrease in ψ_C. This is explained by the fact that with dilution of the mixture by nitrogen the vibrational energy losses near the critical cross section of the nozzle decrease and E_f increases, while the residual energy decreases. With a further decrease in ψ_C ($\psi_C < 0.1$) the function E_r begins to grow, since the quantity α decreases, and the energy E will decline.

The maximum specific energy of emission per unit mass of gas is reached at $\psi_C \approx 0.05$–0.1. In calculating the optimum parameters of a CO GDL required to achieve the maximum values of the emission energy W per unit volume of medium in the resonator we employ the following circumstance. The quantity W is proportional to the pressure p_*,

$$W = \rho_1 E = p_* \Phi(z,\ T_*,\ \bar{t},\ S_0/S_*), \tag{2.1}$$

where ρ_1 is the gas density in the resonator.

An analysis of the function $\Phi(z, T_*, S_0/S_*, \bar{t}) = W/p_*$ (the specific energy from a unit volume at $p_* = 1$ atm), in contrast to the function W, allows one to not increase the number of independent parameters of the problem. The transition from Φ to the optimum values can be accomplished for concrete variants of the calculation.

An analysis of (2.1) shows that for each set of values of the free parameters z and T_* there is a degree of stream expansion S_0/S_* at which the function Φ is maximal, i.e.,

$$\partial \Phi(z,\ T_*,\ \bar{t},\ S_0/S_*)/\partial(S_0/S_*) = 0. \tag{2.2}$$

From the condition (2.2) one can find the optimum values of S_0/S_* as a function of the parameters z and T_*.

The function $\Phi(z, T_*)$ optimized with respect to the parameter S_0/S_* at $\psi_C = 0.1$ and $\bar{t} = 2 \cdot 10^{-7}$ is shown in Fig. 3 in the form of intersecting curves $S_0/S_* =$ const and z = const. The numbers by the lines correspond to the values of S_0/S_* and of z, atm · cm.

It should be noted that a decrease in the carbon monoxide concentration in the range of $0.1 \leq \psi_C \leq 1$ leads to an increase in the specific energy Φ, while the corresponding optimum degree of expansion decreases.

When the parameter T_* is varied with z = const there is a temperature at which the energy Φ is maximal, with Φ_{max} being the larger, the higher T_* and the smaller z.

3. A GDL with a Resonator of Limited Length l_0. In comparing the characteristic time of redistribution of molecules over the vibrational levels with the time of passage of the gas stream through a resonator of length l_0 we find a relation limiting the region of pressures and temperatures of the gas inside the resonator:

$$l(p_1, T) \equiv l(T_*, p_*, S_0/S_*) \leqslant l_0. \tag{3.1}$$

We will seek the maximum of the function E with respect to the three parameters $\bar{t}$, p_*, and S_0/S_*. The parameters T_* and $\delta = h_*/\tan\varphi$ of the problem are kept free. In this case Eq. (1.1) will appear as follows:

$$E(\bar{t}, \bar{a}, \delta, T_*, p_*, S_0/S_*) =$$
$$= \eta(\bar{t}, \bar{a})[E_f(\delta, T_*, p_*) - E_r(T_*, S_0/S_*, p_*, \bar{t}, \bar{a})].$$

An anslysis shows that a maximum of the function E is reached both within the $p_* - S_0/S_*$ region and at the limit (3.1). In the case when the maximum of E is reached at the limit of the $p_* - S_0/S_*$ region it turns out that for the values of the free parameters of the problem under consideration the calculated value of the function E using Eqs. (1.2) and (3.1) exceeds the maximum value of the energy found without the use of (1.2) and (3.1). This result is obtained from an analysis of the simplified kinetics of the processes in the resonator region and needs a more rigorous foundation.

The results of a calculation of the specific emission energy of a CO GDL and of the optimum pressure p_* as a function of the two free parameters T_* and δ are presented in Fig. 4. The numbers by the lines are the values of δ, cm.

The emission energy declines with a decrease in the temperature T_* and with an increase in δ, while the optimum pressure grows with a decrease in temperature.

The corresponding values of the optimum degree of expansion of the nozzle hardly depend on T_* and are equal to $S_0/S_* \approx 200$, 120, 90, and 65 for $\delta = 0.02$, 0.2, 1, and 5, respectively.

The degree of sensitivity of the quantity E to a departure of the parameters p_* and S_0/S_* from the optimum values p_{*op} and $(S_0/S_*)_{op}$ for $T_* = 2000$°K and different δ is illustrated by Fig. 5, where the numbers by the lines are the relative decrease in E, while $\varkappa = S_0/S_*/(S_0/S_*)_{op}$; the maxima of E with respect to the the pressure and the degree of expansion of the nozzle are smooth. The emission energy decreases by 20% when p_* and S_0/S_* are chosen as differing from the optimum values by 5-10 times and by 1.5-2 times, respectively.

In the region of high temperatures and pressure ($T_* > 2500$°K, $p_* > 500$ atm) the calculated energy characteristics of a CO GDL can prove to be overstated, since the present model does not take into account a possible additional channel of deactivation of vibrational energy connected with V–T relaxation on the dissociation products of the initial molecules and in triple collisions.

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